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COMPUTER TECHNIQUE FOR CALCULATIONS OF POTENTIAL DISTRIBUTION I--ETC(U)
SEP 77 K T HUANG, B R MILNER, A W MCCLAINE

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Technical Note

CEL

TN no. N-1502

title: COMPUTER TECHNIQUE FOR CALCULATION OF POTENTIAL DISTRIBUTION IN MULTIDIELECTRIC MEDIA

author: Kwang Ta Huang, Ph.D., Brian R. Milner, and Andrew W. McClaine

date: September 1977

sponsor: Naval Facilities Engineering Command

program nos: YF53.534.011.01.001

CIVIL ENGINEERING LABORATORY
NAVAL CONSTRUCTION BATTALION CENTER
Port Hueneme, California 93043

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) CEL-TN-1502	2. GOVT ACCESSION NO. DN344001	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) COMPUTER TECHNIQUE FOR CALCULATION OF POTENTIAL DISTRIBUTION IN MULTIDIELECTRIC MEDIA.		5. TYPE OF REPORT & PERIOD COVERED Not final; Jul 1974 – Sep 1976
		6. PERFORMING ORG. REPORT NUMBER (9) Technical note Jul 74-Sep 76
7. AUTHOR(s) (10) Kwang Ta Huang, [redacted] Brian R. Milner, [redacted] Andrew W. McClaine		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS CIVIL ENGINEERING LABORATORY Naval Construction Battalion Center Port Hueneme, California 93043		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62760N; YF53.534.011.01.001
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Facilities Engineering Command Alexandria, Virginia 22332		12. REPORT DATE (11) Sept[redacted]77
		13. NUMBER OF PAGES 72 (12) 73 p.
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

DDC JAN 4 1978 F

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

(16) F53534 (17) YF53534011

18. SUPPLEMENTARY NOTES

391 111

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Insulator, dielectric, electric field, equipotential, potential distribution, computer program, computer plotting, Laplace's equation, Poisson's equation.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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20. Continued

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COMPUTER TECHNIQUE FOR CALCULATION OF POTENTIAL DISTRIBUTION IN MULTIDIELECTRIC MEDIA, by Kwang Ta Huang, Ph.D., Brian R. Milner, and Andrew W. McClaine
TN-1502   72 pp illus   September 1977   Unclassified

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INTRODUCTION

The Navy operates and maintains many shore-based communication facilities that employ large insulators in their antenna arrays. Due to various problems with these insulators, an investigation of insulator failures, alternative insulator design, and new concept development was begun.

Electrical difficulties, such as arcing, corona, and heating or burning of insulators, are associated with the presence of an extremely high electric field. Therefore, when evaluating old and new insulator designs, it is helpful to know the electric field distribution associated with each insulator. The geometric complexity of insulators and their associated electric fields precludes the use of classical analytical methods to determine these fields. This report describes a computer program developed to calculate and plot the electric potential distribution of insulator configurations.

Many antenna insulators have geometries that are axially symmetric; that is, the insulator appears the same regardless of how it is turned about its axis of symmetry. The same is true of the electric fields associated with such an insulator. Thus, the problem can be reduced from one involving three dimensions to one of two dimensions.

The program calculates the potential distribution for multidielectric media by numerically solving Laplace's equation. The potential distribution is found in a direct manner by solution of simultaneous, finite-difference equations. Computer drawings of equipotential lines can then be made.

The program is capable of solving many other types of problems, including electric flux lines, mechanical stress, and temperature distribution. Geometries that are strictly two dimensional can be treated by making a minor modification to the program.

This report describes the mathematics behind and organization of the computer program. A detailed users guide is not given here, although one is being prepared. A listing of the computer statements is given in the Appendix.

BACKGROUND

The electric fields of high voltage insulators satisfy Maxwell's equations. But all of Maxwell's equations do not need to be solved in order to obtain the desired information about the insulator fields. In studying the electrical breakdown of insulator configurations, it is the electric field intensity which is of interest. The electric field intensity, E, consists of two components as shown in Equation 1.

$$\bar{E} = - \nabla V - \frac{\partial \bar{A}}{\partial t} \qquad (1)$$

The scalar potential function, V, is related to the voltage applied across the insulator, and the voltage gradient, ∇V, at a given point is related to the dimensions of the insulator and the distance from the electrodes of the insulator. The other component is the rate of change of the magnetic vector potential, $\bar{A}$, which is a function of the current distribution on the antenna, the geometry of the insulator electrodes, and the distance from the insulator electrodes.

Since insulators consist of one or more dielectric materials that are different from air, it is necessary to apply the constituitive relation to Equation 1 to get the displacement flux density, $\bar{D}$.

$$\bar{D} = \varepsilon \bar{E} = \varepsilon \left(- \nabla V - \frac{\partial \bar{A}}{\partial t} \right) \qquad (2)$$

where ε is the dielectric constant of the material in which $\bar{D}$ is evaluated.

In a region that contains no free electric charge, such as the region around the insulator electrodes, Maxwell's divergence equation for the electric field is written as

$$\nabla \bullet \bar{D} = 0 \qquad (3)$$

Applying this condition to Equation 2 yields

$$\nabla \bullet \bar{D} = - \varepsilon \left(\nabla \bullet \nabla V + \nabla \bullet \frac{\partial \bar{A}}{\partial t} \right) = 0 \qquad (4)$$

or

$$\nabla \bullet \bar{D} = - \varepsilon \left[\nabla^2 V + \frac{\partial}{\partial t} (\nabla \bullet \bar{A}) \right] = 0 \qquad (5)$$

$\nabla \bullet \bar{A}$ is defined by the Lorentz condition, Equation 6,

$$\nabla \bullet \bar{A} + \mu \varepsilon \frac{\partial V}{\partial t} + \mu \sigma V = 0 \qquad (6)$$

where μ and σ are the permeability and conductivity of the medium, respectively. Using Equation 6 to substitute for $\nabla \bullet \bar{A}$, Equation 5 becomes

$$\nabla \bullet \bar{D} = -\varepsilon \left[\nabla^2 V - \frac{\partial}{\partial t} \left(\mu \varepsilon \frac{\partial V}{\partial t} + \mu \sigma V \right) \right] = 0 \tag{7}$$

or

$$\nabla \bullet \bar{D} = -\varepsilon \left(\nabla^2 V - \mu \varepsilon \frac{\partial^2 V}{\partial t^2} - \mu \sigma \frac{\partial V}{\partial t} \right) = 0 \tag{8}$$

Since V, in the steady-state operating condition, is of the form

$$V = V_o(r, z) \, e^{j\omega t} \tag{9}$$

at any given point in space for some $V_o(r, z)$, the time derivatives of V are given by

$$\frac{\partial V}{\partial t} = j \omega V \tag{10}$$

$$\frac{\partial^2 V}{\partial t^2} = -\omega^2 V \tag{11}$$

where ω is the angular frequency of the system. Table 1 shows approximate values for the constants in Equation 8 for the space between the electrodes of most types of VLF antenna insulators.

Table 1. Constants for Regions Near Electrodes of Most VLF Antenna Insulators

Constant	Symbol	Approximate Value
Angular frequency	ω	2×10^5 sec^{-1}
Dielectric constant	ε	1×10^{-11} F/m
Permeability	μ	1×10^{-6} H/m
Conductivity	σ	1×10^{-11} (ohm-m)$^{-1}$

Rewriting Equation 8 using Equations 10 and 11

$$\nabla \bullet \bar{D} = -\varepsilon \left[\nabla^2 V + (\mu \varepsilon \omega^2 - j \mu \sigma \omega) V \right] = 0 \tag{12}$$

But

$$\mu\,\varepsilon\,\omega^2 \;-\; j\,\mu\,\sigma\,\omega \;\simeq\; 4 \times 10^{-7} \;-\; j\,2 \times 10^{-12} \;\simeq\; 4 \times 10^{-7} \tag{13}$$

This implies that the $j\mu\sigma\omega V$ term may be ignored when considering the steady-state voltage of most insulators, since it is much smaller than the $\mu\varepsilon\omega^2$ term. Equation 12 then becomes

$$\nabla \bullet \overline{D} \;=\; -\,\varepsilon(\nabla^2 V \;+\; k^2\,V) \;=\; 0 \tag{14}$$

where

$$k^2 \;=\; \mu\,\varepsilon\,\omega^2 \;\simeq\; 4 \times 10^{-7} \tag{15}$$

For the purpose of evaluating Equation 14 it is sufficient to approximate the VLF antenna insulator as a parallel-plate capacitor shown in Figure 1. In the cylindrical coordinate system Equation 14 is written as

$$-\,\varepsilon\left[\frac{1}{r}\left(\frac{\partial}{\partial r}\right)\left(r\,\frac{\partial V}{\partial r}\right) \;+\; \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) \;+\; \frac{\partial^2 V}{\partial z^2} \;+\; k^2\,V\right] \;=\; 0 \tag{16}$$

Since the problem has cylindrical symmetry, V is independent of ϕ and $\partial^2 V/\partial\phi^2 = 0$. To facilitate easy analysis, the problem can be simplified by considering the region near the middle of the capacitor, far from the edges. In this region, V is independent of r, and $\partial V/\partial r = 0$. Equation 14 then becomes:

$$\frac{\partial^2 V}{\partial z^2} \;+\; k^2\,V \;=\; 0 \tag{17}$$

This equation has a simple solution which can be easily analyzed.

$$V \;=\; \frac{V_o}{\sin k\,d}\,\sin k\,z \tag{18}$$

For a VLF insulator

$$k \;\simeq\; 6 \times 10^{-4}\ \text{meter}^{-1}$$

$$d \;\simeq\; 1\ \text{meter}$$

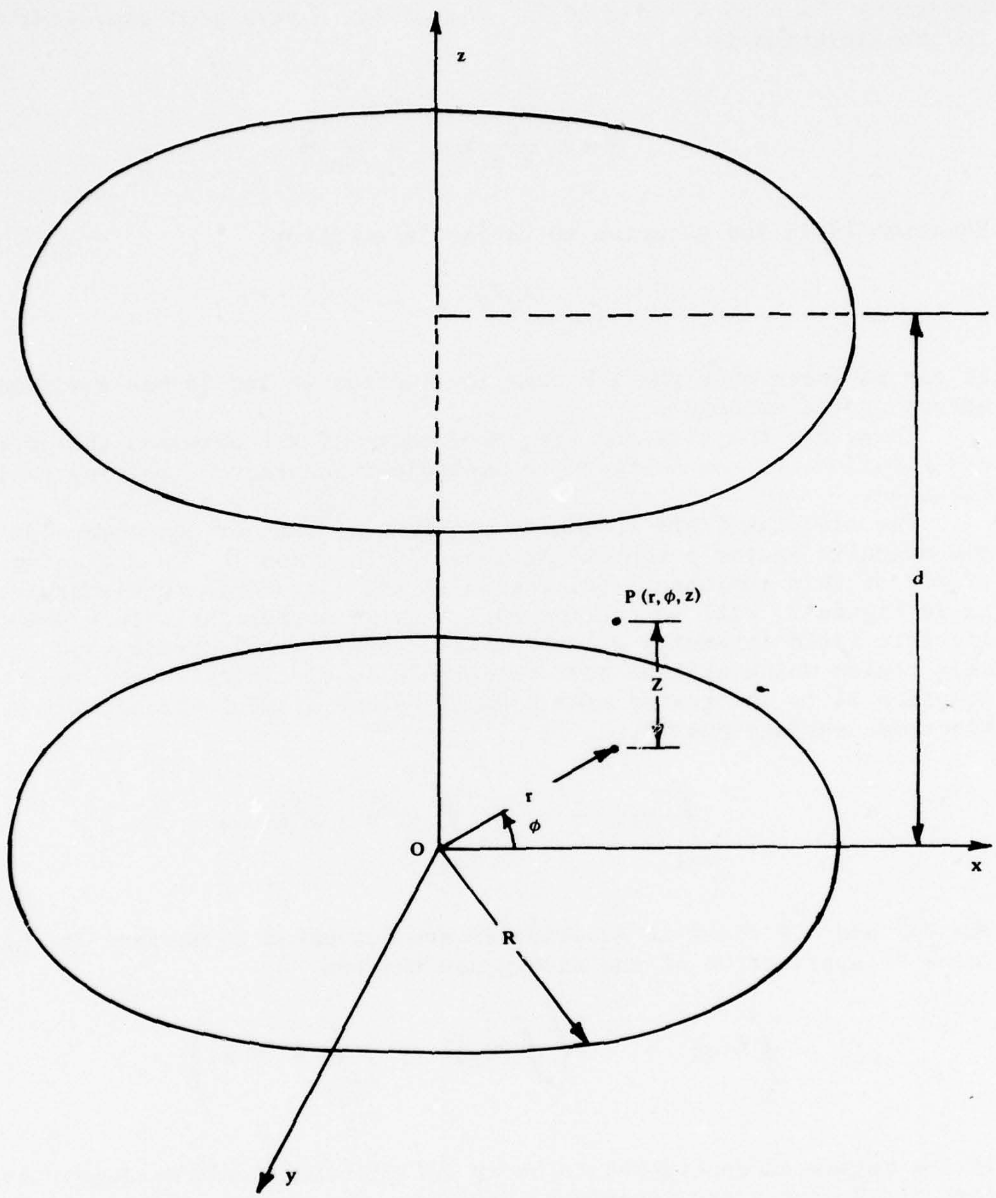


Figure 1. Parallel-plate capacitor.

But sin $x \simeq x$ when $x < 5 \times 10^{-2}$. Therefore, a very good approximation for the solution is

$$V = \frac{V_o}{k\,d}\,k\,z = V_o\,\frac{z}{d} \tag{19}$$

Equation 19 is the solution to Laplace's equation:

$$\nabla^2 V = 0 \tag{20}$$

It can be shown that the k^2V term in Equation 14 and 16 has a minimal effect on the solution.

Thus, for the size and frequency range of VLF antennas the potential distribution, V, can be found to sufficient accuracy by solving Laplace's equation.

The electric field intensity consists of another component due to the magnetic vector potential as shown in Equation 1. In analyzing the effect of this component, the region at the surface of an electrode, as in Figure 2, will be considered. This is appropriate since the electric field intensity will be maximum near the electrode, and it is this region which will be most vulnerable to electrical breakdown. Equation 14 is integrated over a small volume element through which the electrode surface passes.

$$\int_{vol} \nabla\bullet\bar{D} = -\,\varepsilon \int_{vol} (\nabla^2 V + k^2 V)\,dv \tag{21}$$

The $\nabla\bullet\bar{D}$ and $\nabla^2 V$ terms in Equation 21 are converted to surface integral terms by application of the divergence theorem.

$$\oint_s \bar{D}\bullet d\bar{s} = -\,\varepsilon\left(\oint_s \nabla V\bullet d\bar{s} + \int_{vol} k^2\,V\,dv\right) \tag{22}$$

If the region is considered to be an infinitesimal volume element so that $\bar{D}$, ∇V, and V are invariant throughout the region, the equation becomes

$$\bar{D}_o\bullet\oint_s d\bar{s} = -\,\varepsilon\left(\nabla V_o\bullet\oint_s d\bar{s} + k^2\,V_o\int_{vol} dv\right) \tag{23}$$

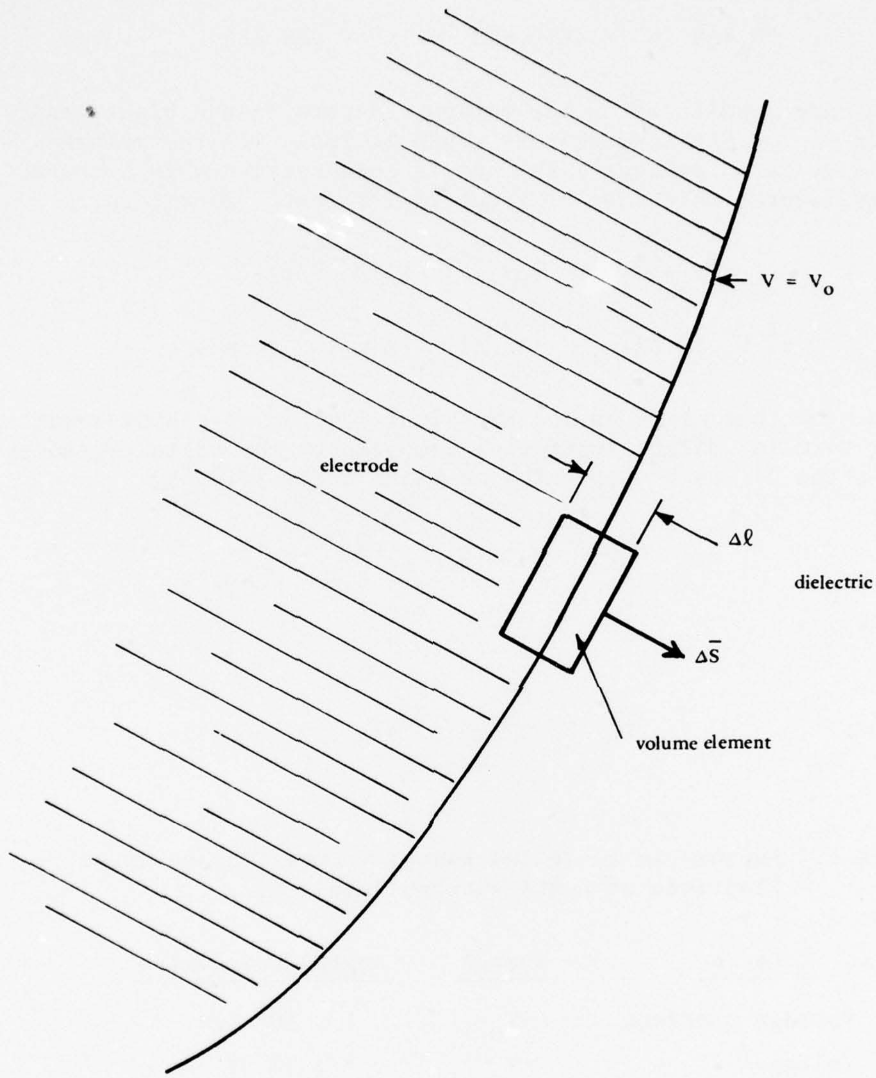


Figure 2. Cross section of electrode and volume element.

where $\bar{D}_o$, ∇V_o, V_o are values at the surface of the electrode. Completing the integration yields:

$$\bar{D}_o \bullet \Delta\bar{S} = -\varepsilon(\nabla V_o \bullet \Delta\bar{S} + k^2 V_o \Delta S \Delta\ell) \tag{24}$$

The approximate magnitudes of the various factors in the right-hand side of Equation 24 for VLF antennas are given in Table 2. The voltage gradient value is an average value and is conservative when compared to the peak gradients, which are of greatest interest. Therefore,

$$|\nabla V_o \bullet \Delta\bar{S}| \simeq |\Delta S| \ (1 \times 10^5 \text{ V/m})$$

$$|k^2 V_o \Delta S \Delta\ell| \simeq |\Delta S \Delta\ell| \ (2 \times 10^{-1} \text{ V/m}^2)$$

Keeping in mind that $\Delta\ell$ is an infinitesimal length, it can be seen that the $k^2 V_o$ term is insignificant when compared to the voltage gradient term. Equation 24 can be approximated quite accurately by

$$\bar{D}_o \bullet \Delta\bar{S} \simeq -\varepsilon \nabla V_o \bullet \Delta\bar{S} \tag{25}$$

or, simplifying

$$\bar{E}_o = \frac{\bar{D}_o}{\varepsilon} \simeq -\nabla V_o \tag{26}$$

Table 2. Factors in an Infinitesimal Volume Element at an Electrode of a VLF Antenna Insulator

Factor	Symbol	Approximate Value
Voltage gradient	∇V_o	1×10^5 V/m
Voltage	V_o	5×10^5 V
Wave number	k	6×10^{-4} m^{-1}

In summarizing, the electric field intensity near the electrodes of an insulator is very nearly equal to the voltage gradient at that point; the component contributed by the magnetic vector potential in Equation 1 can be neglected. The electric field intensity can be interpreted from

the potential distribution by Equation 26. For example, if equipotential lines are plotted, the regions where the lines are most dense are the regions of highest potential gradient and highest electric field intensity. It is easier to visualize the electric fields of insulators in this manner than by plotting electric field lines themselves. For this reason, the computer program is designed to calculate and plot the potential distribution.

COMPUTER SOLUTION METHOD

Potential Distribution Problem

As described above, the potential distribution near an insulator can be found accurately by solving Laplace's equation (Equation 20). Laplace's equation can be rewritten in the form of Equation 14, ignoring the k^2V term, since its effect on the solution is negligible:

$$\varepsilon(\nabla^2 V) = \nabla \bullet \bar{D} = 0 \tag{27}$$

Coordinate Grid System. Finite-difference techniques are used to solve the equation numerically. The geometry of the region near the antenna insulator is described according to a grid system, such as the one shown in Figure 3. The grid is drawn in a plane passing through the axis of symmetry of the insulator so that a section view of the insulator can be defined in terms of the grid points. The boundary of a material (electrode or dielectric) is approximated by a series of line segments that connect adjacent grid intersection points. These line segments can be vertical, horizontal, or diagonal. All line segments must start and end on grid intersection points, and diagonals must connect two points that are formed by the intersection of adjacent horizontal and vertical grid lines. By properly choosing the spacing of grid lines and using combinations of horizontal, vertical, and diagonal line segments, the material boundaries of an insulator can be described quite accurately.

The ability to use variable grid density is an important feature. If the entire grid region were composed of equally spaced grid lines, the total number of grid lines would be determined by the density required to accurately describe the smallest feature of the insulator configuration. This often results in an excessively large number of grid lines in regions where they are not necessary. Allowing a variable grid density makes it possible to use a high grid density only in regions where it is necessary to achieve an accurate description of the insulator or where detailed information on the potential distribution solution is required. The result is a substantial savings in computer memory requirements and program execution time. Also, if a uniform grid size is used, it is often necessary to compute two or more solutions, using a finer grid with each new solution in order to obtain the desired accuracy in a given region of interest. When using a variable grid density, one solution is usually all that is required.

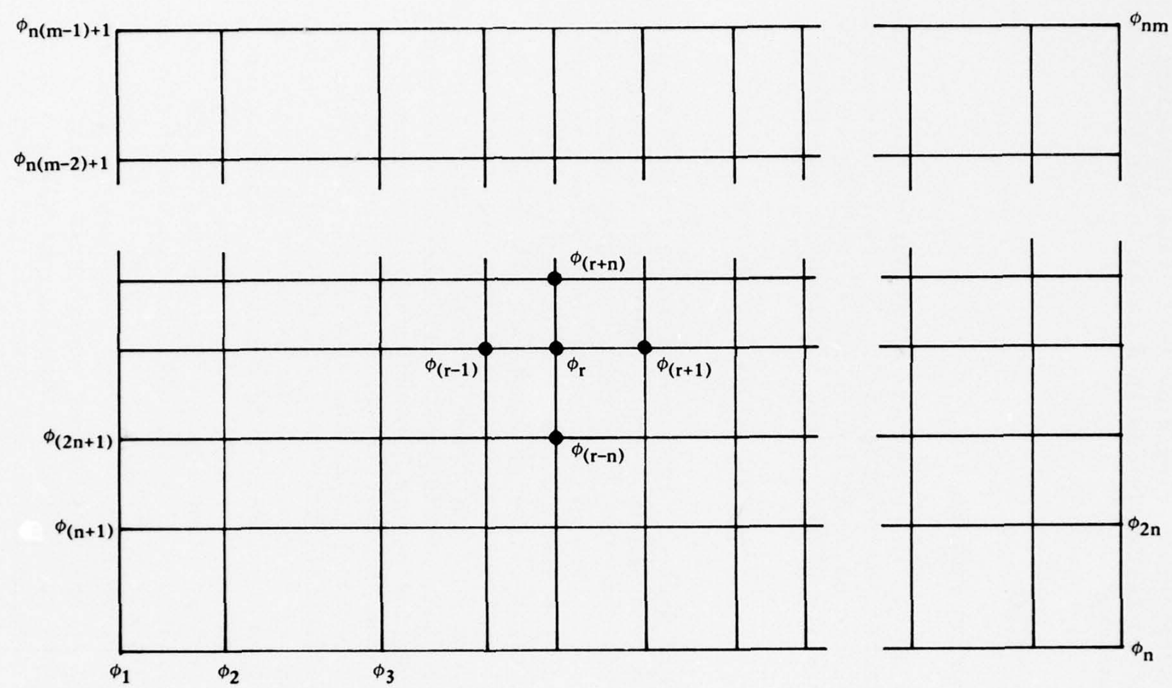


Figure 3. Grid system.

Formulation of Numerical Equations. Equation 27 is integrated over a volume element surrounding each grid intersection as shown in Figure 4.

$$\int_{vol} \varepsilon \, \nabla^2 V \, dv = \int_{vol} \overline{\nabla} \bullet \overline{D} \, dv = 0 \tag{28}$$

A surface integral is obtained by applying the divergence theorem to Equation 28.

$$\int_{vol} \overline{\nabla} \bullet \overline{D} \, dv = \oint_{s} \overline{D} \bullet \overline{ds} = \oint_{s} \varepsilon \, \nabla V \bullet \overline{ds} = 0 \tag{29}$$

This is an integral of the displacement flux densities crossing the surface areas of the volume element. Equation 29 is applied to the surfaces of the volume element in Figure 4. Due to cylindrical symmetry, the voltage gradient in the ϕ direction is zero. Thus, the integrals over the vertical faces in the ϕ-plane are also zero. The gradients across the remaining four faces are generally not zero. These remaining faces are chosen such that they pass through the midpoints of the line segments joining the central point, whose potential is ϕ_o, and the corresponding horizontally or vertically adjacent point, ϕ_1 through ϕ_4. Each of the four faces can be divided by a boundary which separates two different dielectric materials. There are, then, a total of eight possible dielectrics and eight different homogeneous segments through which the flux, $q_{i,j}$, may pass.

In calculating the integral in Equation 29 numerically, the flux crossing the surfaces is approximated by a finite difference value for the voltage gradient. The normal component of flux passing through a given face is assumed to be uniform across that face. For example,

$$q_{2,3} = \frac{\phi_2 - \phi_o}{h_2} \tag{30}$$

Since this normal component of the voltage gradient is perpendicular to the surface through which it passes, the dot product in Equation 29 is reduced to simple multiplication. The integral in Equation 29, then, is the sum of eight terms, each of which is a product of a displacement flux density and the area through which the flux passes. The form in which this equation is used in the program, with the coefficients of the ϕ's grouped together and a factor of π dropped, is

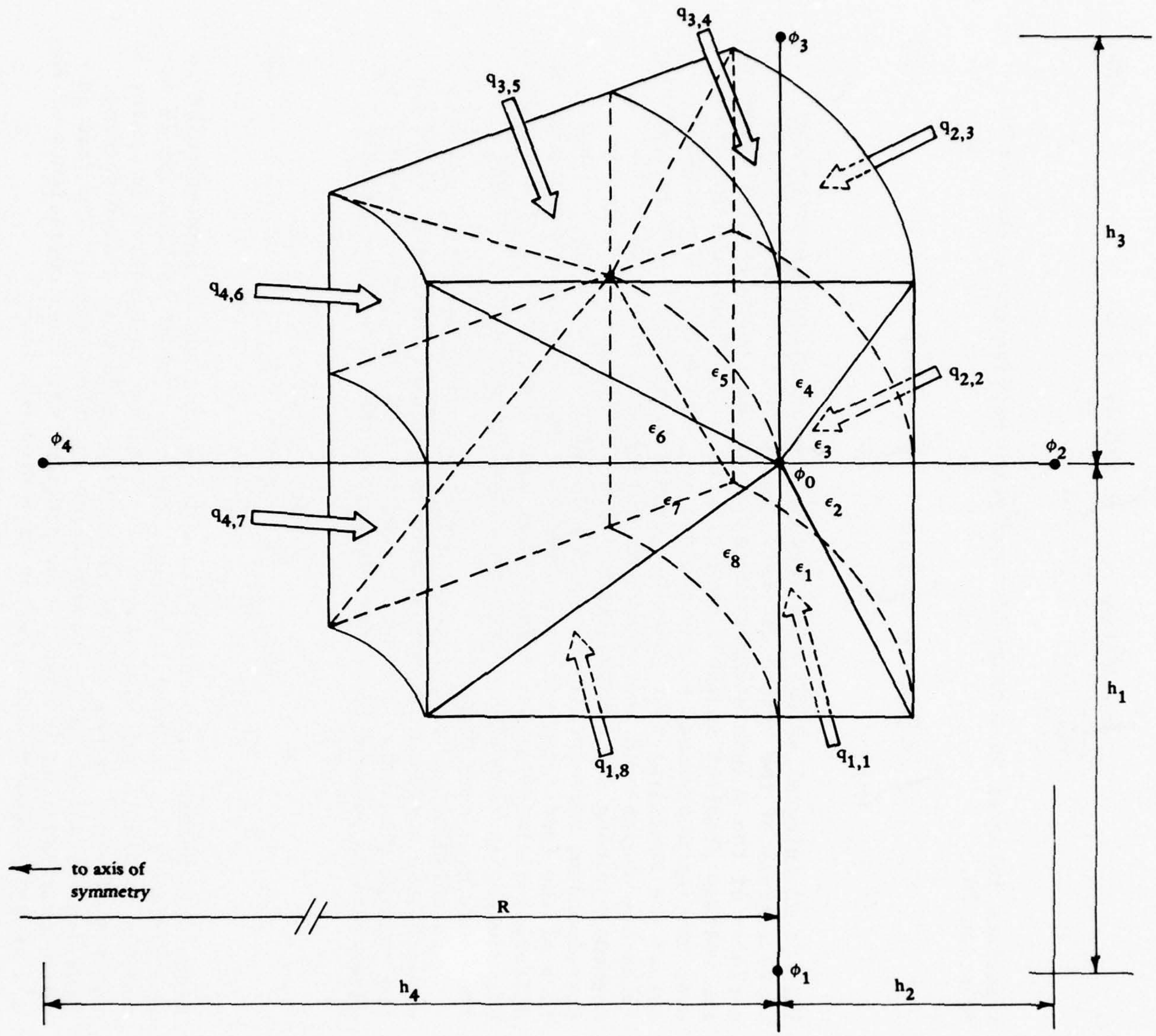


Figure 4. Volume element.

$$\frac{\phi_1}{h_1}\left[\varepsilon_1 h_2\left(R+\frac{h_2}{4}\right) + \varepsilon_8 h_4\left(R-\frac{h_4}{4}\right)\right]$$

$$+\frac{\phi_2}{h_2}\left[(\varepsilon_3 h_3 + \varepsilon_2 h_1)\left(R+\frac{h_2}{2}\right)\right]$$

$$+\frac{\phi_3}{h_3}\left[\varepsilon_5 h_4\left(R-\frac{h_4}{4}\right) + \varepsilon_4 h_2\left(R+\frac{h_2}{4}\right)\right]$$

$$+\frac{\phi_4}{h_4}\left[(\varepsilon_6 h_3 + \varepsilon_7 h_1)\left(R-\frac{h_4}{2}\right)\right]$$

$$-\ \phi_o\left[h_4\left(\frac{\varepsilon_8}{h_1}+\frac{\varepsilon_5}{h_3}\right)\left(R-\frac{h_4}{4}\right) + \frac{1}{h_4}(\varepsilon_6 h_3 + \varepsilon_7 h_1)\left(R-\frac{h_4}{2}\right)\right.$$

$$\left. +\ h_2\left(\frac{\varepsilon_1}{h_1}+\frac{\varepsilon_4}{h_3}\right)\left(R+\frac{h_2}{4}\right) + \frac{1}{h_2}(\varepsilon_3 h_3 + \varepsilon_2 h_1)\left(R+\frac{h_2}{2}\right)\right] = 0 \qquad (31)$$

Organization of Equations. There is one equation of the form of Equation 31 for each grid intersection point. If the grid consists of m horizontal and n vertical lines, as in Figure 3, there will be mn = m x n equations. These simultaneous equations can be written in matrix form as

$$A\ \Phi = B \qquad (32)$$

where A is an mn by mn constant matrix, Φ is a vector of mn unknowns, and B is an mn-vector of constants. By carefully choosing the order of the mn equations, A can be made a banded, symmetric matrix as shown schematically in Figure 5. The ordering of the equations is indicated by the subscripts of the ϕ's in Figure 3. The r^{th} equation, whose coefficients appear in the r^{th} row of the A matrix, involve at most five grid points whose subscripts fall between r - n and r + n, inclusive. This means that the bandwidth of A is never larger than 2n + 1, with n elements on each side of the diagonal. Furthermore, this arrangement scheme yields a symmetric matrix so that only the diagonal elements and the elements on one side of the diagonal need to be stored in the computer in order to know the entire matrix. That means storage for only mn x (n + 1) elements is required instead of mn x mn if all elements were needed. This ordered numbering system results in a substantial savings in computer storage. The bandwidth is narrowest and memory

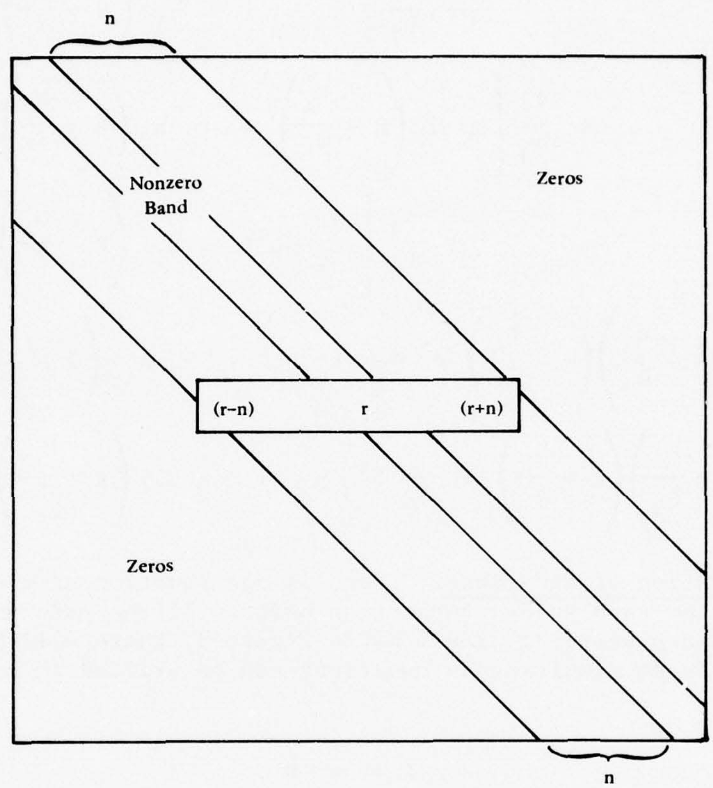


Figure 5. Symmetrical, banded matrix.

requirements smallest when n is less than m. Such is the case for most axially symmetric antenna insulators, which tend to have diameters that are smaller than their lengths. This usually results in the need for fewer vertical grid lines than horizontal grid lines. Consequently, the computer program is arranged such that the vertical grid lines are parallel to the axis of symmetry.

There are few nonzero elements in B. These nonzero elements correspond to equations which involve grid points that are of a known specified potential. For example, if a given ϕ_r is a point on an electrode of potential V_o, Equation 31 is not used, and the equation for that point is simply

$$\phi_r = V_o \tag{33}$$

where V_o becomes an element of the B vector.

Solution of Equations. Once all of the mn equations are formulated, the matrix equation (Equation 32) is converted to reduced echelon form by the process of Gauss-Jordan elimination. The ϕ solutions are then calculated by back substitution into the reduced set of equations.

Boundary Conditions. There are several grid points that must be treated specially. One of these, in which a grid intersection is a point on a conductor or electrode of known potential, has already been discussed. Points along the edge of the grid must also be given special consideration. If the potentials along the grid boundary are known, those points can be treated as described above for Equation 33. If the grid boundary potentials are unknown, other methods must be used since these boundary points have only two or three neighboring points instead of four as in the general case of Figure 4.

For a grid boundary that coincides with the axis of symmetry, there are four grid points involved as in Figure 6. The same arguments apply as for Figure 4, except that there are only four dielectric segments through which flux passes. The displacement flux density through each segment is multiplied by the surface area through which the flux passes. With the sum of these four terms set equal to zero, a factor of π dropped, and the coefficients of the ϕ's grouped together, an equation similar to 31 appears that corresponds to Equation 29.

$$\frac{\phi_1}{h_1}\left(\varepsilon_1 \frac{h_2^2}{4}\right) + \frac{\phi_2}{h_2}\left(\varepsilon_2 \frac{h_1\, h_2}{2} + \varepsilon_3 \frac{h_2\, h_3}{2}\right) + \frac{\phi_3}{h_3}\left(\varepsilon_4 \frac{h_2^2}{4}\right)$$

$$- \phi_o\left(\varepsilon_1 \frac{h_2^2}{4\, h_1} + \varepsilon_2 \frac{h_1}{2} + \varepsilon_3 \frac{h_3}{2} + \varepsilon_4 \frac{h_2^2}{4\, h_3}\right) = 0 \tag{34}$$

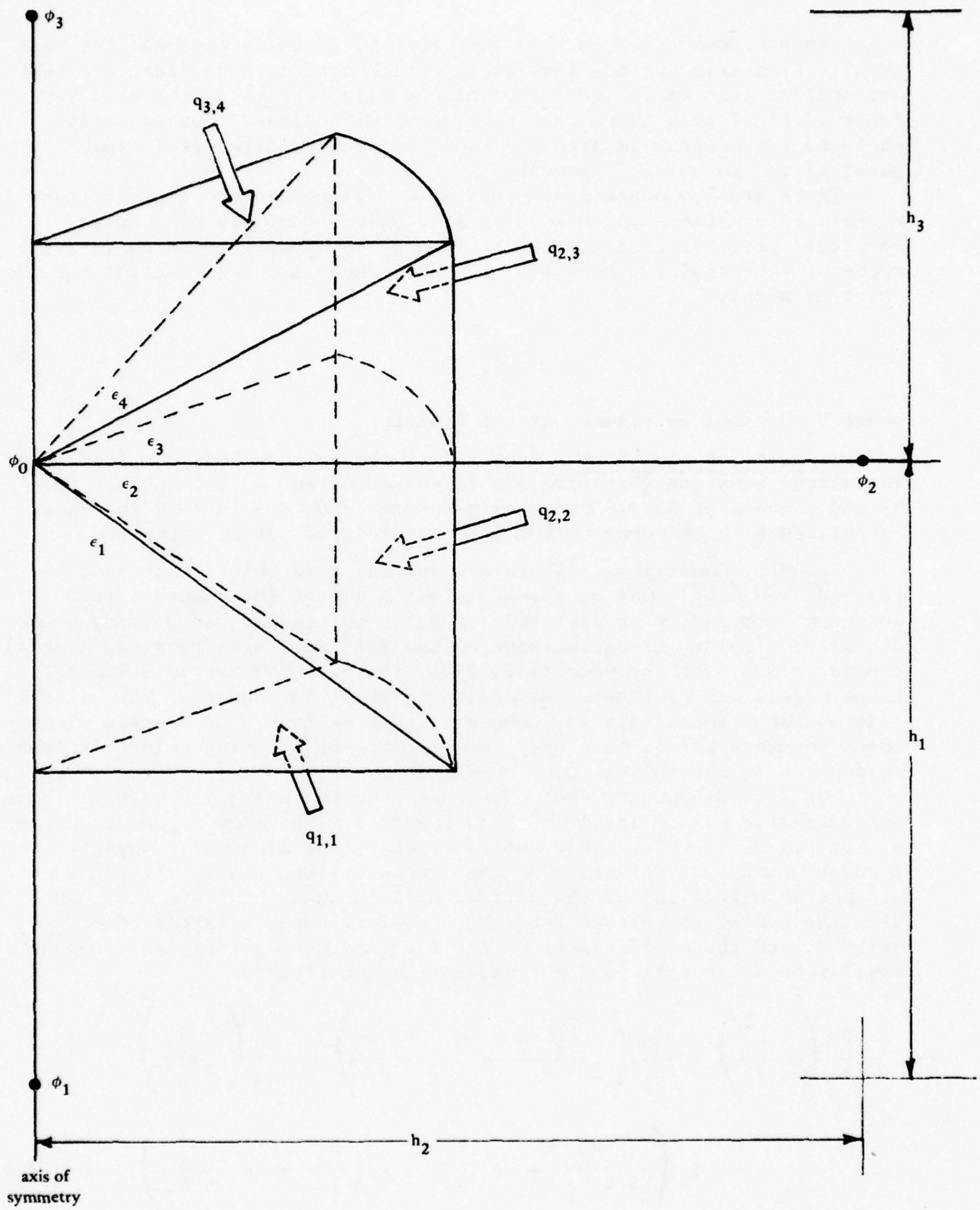


Figure 6. Volume element along axis of symmetry.

In many problems the bottom horizontal grid line represents a plane of symmetry. For example, if the two ends of a guy line insulator are mirror images of each other, the plane which passes through the midpoint of the insulator and is perpendicular to the axis of symmetry is a plane of symmetry. The method of images can be applied to simplify the problem and obtain the boundary condition. Since both ends of the insulator are identical, the solution needs only to be calculated from the plane of symmetry toward one end. The boundary condition at the plane of symmetry can be satisfied by assigning all points on the corresponding grid line a potential equal to the value midway between the potentials of the two insulator ends.

In all cases where an explicit boundary potential is unknown, another means of satisfying the boundary condition requirements must be found. A simple method of obtaining the boundary condition is to determine an approximation for a particular electric flux line that exists at a large distance from the region of greatest interest. Figure 7 depicts such a flux line for a guyline insulator which is not symmetrical about its midpoint. The lack of end-to-end symmetry of the insulator assembly requires that three of the grid boundaries (top, bottom, and right-hand sides) must be treated in this manner. The location of the distant flux line can be found easily by analytical methods, since most field problems degenerate to a very simple problem when only the solution at a distant point is required. In the case of a guyline insulator, the assembly appears electrically as a simple needle gap when viewed from a large distance. The needle gap problem is one for which the fields are easily calculated. A flux line is by definition a line of direction of the electric field. The flux is along and parallel to the line, and no flux crosses the line. If the grid intersection labeled ϕ_o in Figure 4 were located on or inside this flux line, it would experience no displacement flux density from outside the flux line. This can be described mathematically by assigning $\varepsilon = 0$ for the region outside the flux line. There is no known material having a dielectric constant of zero; this is merely a mathematical technique that effectively satisfies the boundary condition. As a result, the points beyond the flux line, in the region where the dielectric constant is zero, have no bearing on the potential solution in the interior region, and it is not necessary to know the potentials along the outer grid boundary.

Treatment of Conductors. Conductors are fundamentally different from dielectrics and must be given special treatment. If the potential is known, as with insulator electrodes, the grid points in that conductor are treated as in Equation 33. But some insulator assemblies have metal components that are between the electrodes and are insulated from any fixed potential. An example of such a "floating potential conductor" is the pin that is used to connect one or more suspension insulators together to form a chain of insulators. The potential of such a conductor is not known and cannot be specified on input; it must be calculated during the execution of the program. This is done by using the same method as for other points, but a very high dielectric constant is used to describe

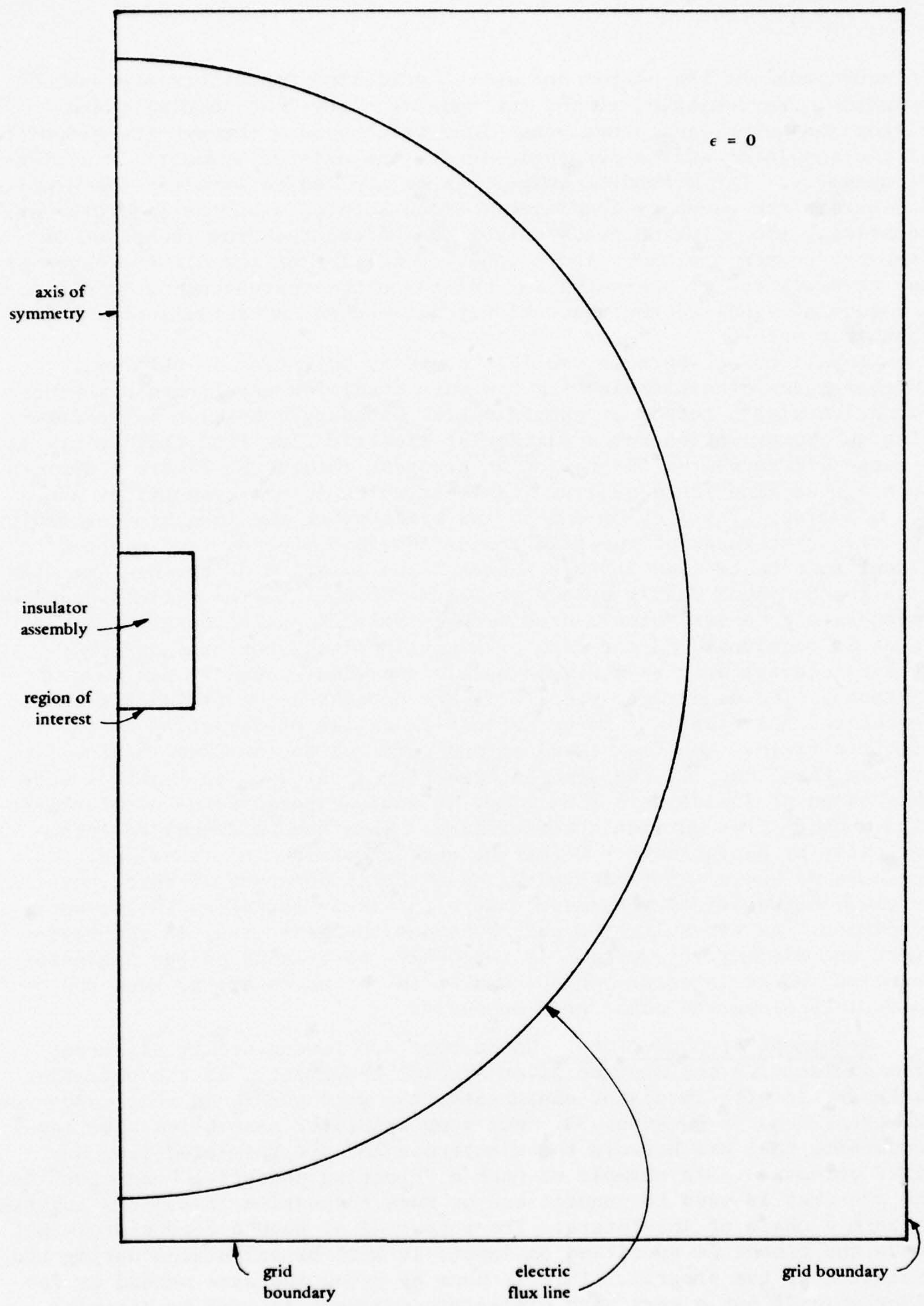


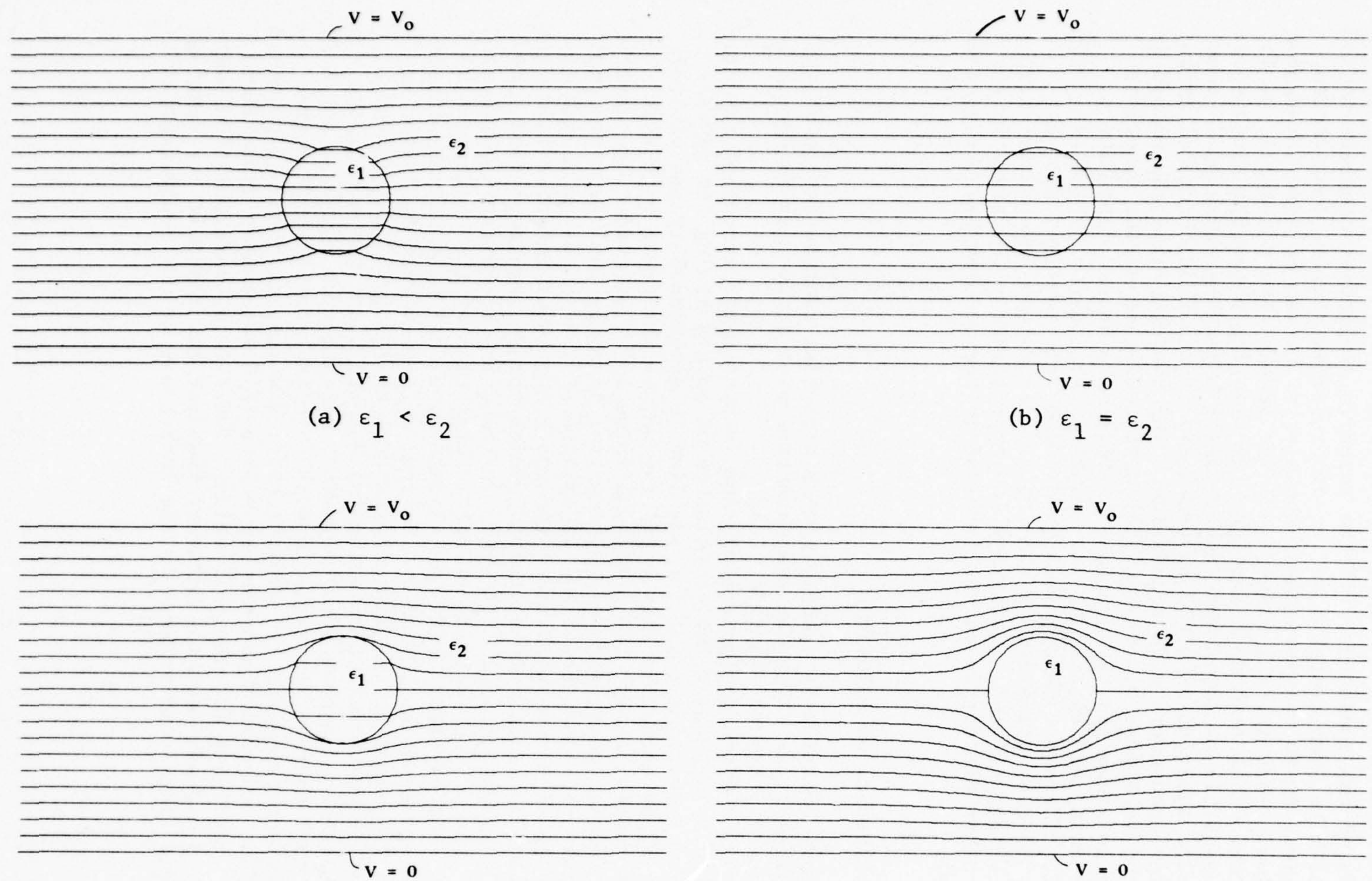
Figure 7. Boundary condition at a distant electric flux line.

the region of the conductor. If the relative dielectric constant is nearly infinite when compared to the other dielectrics in the problem, it is equivalent to requiring that the electric field be zero throughout that region. That is, all points would be at the same potential, as in a conductor. This method of representing floating potential conductors makes it possible to solve the problem using Laplace's equation instead of Poisson's equation as would be necessary if the free charge distribution on the conductors were considered. The theory behind the method is illustrated in Figure 8. As the dielectric constant of the sphere is increased in views (a) through (d), it begins to appear more like a conductor until, in Figure 8d, all points in the region are at the same potential. The exact magnitude of the dielectric constant that is necessary to give satisfactory results may depend upon the particular computer being utilized. Using a value approximately 10^8 times larger than the other dielectric values involved in the problem has shown to be successful.

Computer Plotting of Solutions

Calcomp plotting is used to give a pictorial representation of the equipotential lines. From an equipotential plot it is easy to determine the regions of highest electric field.

To plot a given equipotential line, a systematic search of the grid is executed to determine the coordinates of points having the given potential. A starting point for the line is determined by successively searching along each grid line until two points are found such that their potentials bracket the potential value to be plotted. The search for this initial point on the equipotential line may include horizontal or vertical grid lines or both. An example of a vertical grid line is shown in Figure 9, where points A through F represent intersections with horizontal grid lines. If a starting point for the equipotential line of value 50.0 were desired, it would be found in the interval between points C and D. The location of the intersection of the equipotential line with this vertical grid line is determined approximately by interpolation between points C and D. Several methods of interpolation may be used. The simplest is first degree or linear interpolation, which assumes a linear variation in potential between C and D. A slightly more sophisticated method involves using the data from points B and E in addition to C and D. It is then possible to generate a third degree polynomial approximation for the potential distribution between points B and E. This polynomial can then be solved to determine an approximate vertical coordinate for the 50.0 value. The type of interpolation that is employed depends upon the accuracy required and the relative spacing of the grid lines with respect to the smoothness of the equipotential curves.



(a) $\varepsilon_1 < \varepsilon_2$

(b) $\varepsilon_1 = \varepsilon_2$

(c) $\varepsilon_1 > \varepsilon_2$

(d) $\varepsilon_1 >> \varepsilon_2$

Figure 8. Equipotential line plots illustrating the effect of a very large dielectric constant.

Figure 9. Section of vertical grid line with potential solution at intersection points.

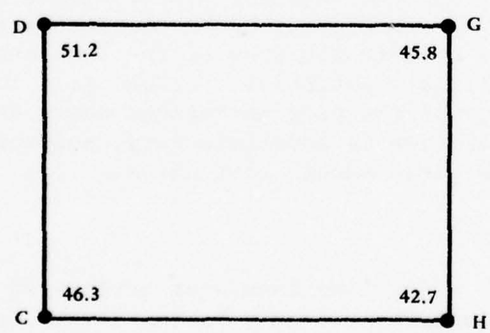


Figure 10. Sample grid rectangle and potential solution at intersection points.

Once an initial point of the equipotential line is obtained, the next point is found by checking the remaining sides of the rectangle formed by adjacent grid lines on one side of the line segment CD, as shown in Figure 10. Since an equipotential line must be continuous throughout the grid region, if it enters a rectangle through one side, such as CD, it must then exit through one of the other three sides. This assumes that the grid network is fine enough so that the equipotential lines cannot enter and exit on the same side of a rectangle. This requirement is satisfied if the grid is designed by the user to be fine enough in regions where the electric field is sharply curved or contorted. Subsequent equipotential points are found by searching successive rectangles. For example, in Figure 10 the next rectangle to be searched would be the rectangle above line segment DG. If the desired potential occurs exactly at a corner of a rectangle, the potential of that grid intersection is shifted by a finite, but small, increment. This avoids numerical difficulties that would occur if the equipotential line were to go through a corner of a rectangle instead of one of the sides. The incremental shift is negligible so that the appearance of the plot is unaffected.

The search process is repeated so that the equipotential is traced throughout the entire grid until the line either ends at the grid boundary or circles back to the initial point. The equipotential curves are drawn by connecting these points.

There are several formats in which the equipotential plot can be drawn. The equipotential lines can be plotted thoughout the entire grid region, or small segments of the grid can be enlarged and plotted. The half-section view of one side of the axis of symmetry can be "rotated" about the axis of symmetry so that a full section view of the insulator can be plotted in one figure. Such a plot, showing the insulator on both the right- and left-hand sides of the axis of symmetry is often easier to interpret than if only one side were plotted. This can be taken a step farther if the insulator assembly to be plotted has end-to-end symmetry so that the two ends are mirror images about a plane of symmetry. The view of one end may be "reflected" across the plane of symmetry such that a full-length view of the insulator is plotted. The method used to interpolate potentials between grid intersections will affect the appearance of the plot in regions where grid lines are sparse. Where linear interpolation is unsatisfactory, polynomial interpolation is often adequate to yield smooth plot curves.

Examples

Figure 11 shows a guy line insulator having end-to-end symmetry. The potential distribution need only be calculated for one-fourth of the section view shown in Figure 11. The solution for the remaining three quadrants can be obtained from the first quadrant solution using symmetry.

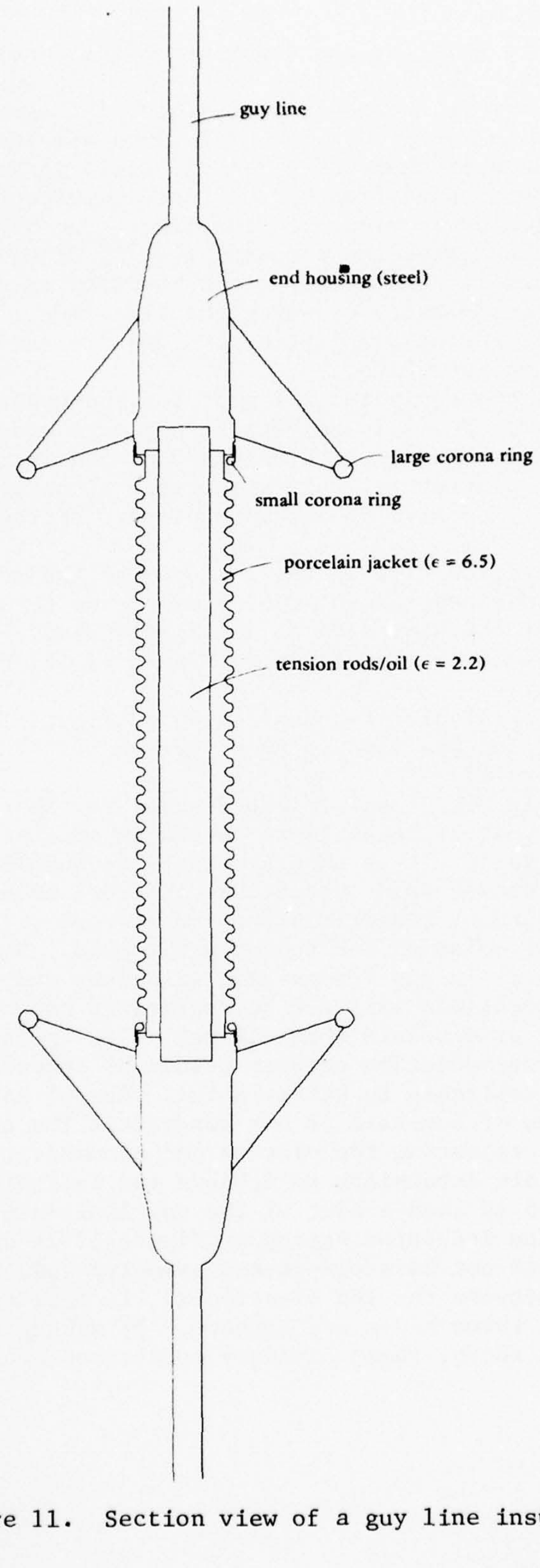


Figure 11. Section view of a guy line insulator.

The coordinate grid for one quadrant of the insulator assembly is shown in Figure 12. The grid lines are very dense near the origin (where the insulator is located), since this is the region where the greatest accuracy is required. The grid lines are less dense in regions away from the insulator where the electric field is smooth. The distant boundary condition is satisfied by the curved dielectric boundary which is an approximation of an electric flux line. The dielectric constant is unity everywhere inside the boundary that is occupied by air. The dielectric constant is zero outside this boundary to simulate the lack of displacement flux density crossing the flux line. The boundary conditions for the bottom and left-hand sides are satisfied by the symmetry of the configuration.

Figure 13 is the equipotential plot for the quadrant of the insulator shown in Figure 12. The voltage between adjacent equipotential lines is 2.5% of the voltage between the electrodes of the insulator in Figure 11. In general, the equipotential values that are to be plotted can be chosen as necessary to give an adequate picture of the potential distribution.

If a more detailed view of the region near the insulator (or any other region) is desired, an enlarged view can be obtained. One way to do this is to plot the same data on a larger scale. Figure 14 is the grid in the region of the insulator in Figure 12 which has been plotted on a larger scale.

The equipotential plot for the region of Figure 14 is given in Figure 15. This plot was made using a portion of the same data that was used to plot Figure 13.

If the grid is not of sufficient density in the region of the insulator, it may not be possible to obtain an accurate enlarged plot from that set of data. It is then necessary to obtain another potential solution using a denser grid system that includes only the region to be enlarged. The boundary conditions for this second grid can be obtained from the potential solution for the original grid. Not all of the boundary grid points in the denser grid will have existed in the coarse grid and their potentials will not be explicitly known. The potentials of those boundary grid points that did not exist in the original grid can be found by interpolation between points of known potential.

It is often desirable to obtain a full view of an insulator plot, rather than a view of one-half or one quadrant. The additional views can be plotted by repeating the plotter pen commands two or four times with the appropriate inversions to achieve the full view of the desired region. Figure 16 is such a plot of the guy line insulator.

The suspension insulator string in Figure 17 is an example of an insulator that does not have end-to-end symmetry and, hence, no plane of symmetry midway between the end electrodes. In this problem, the boundary conditions on three sides are unknown. By making an approximation for the flux line shown, these boundary conditions are satisfied.

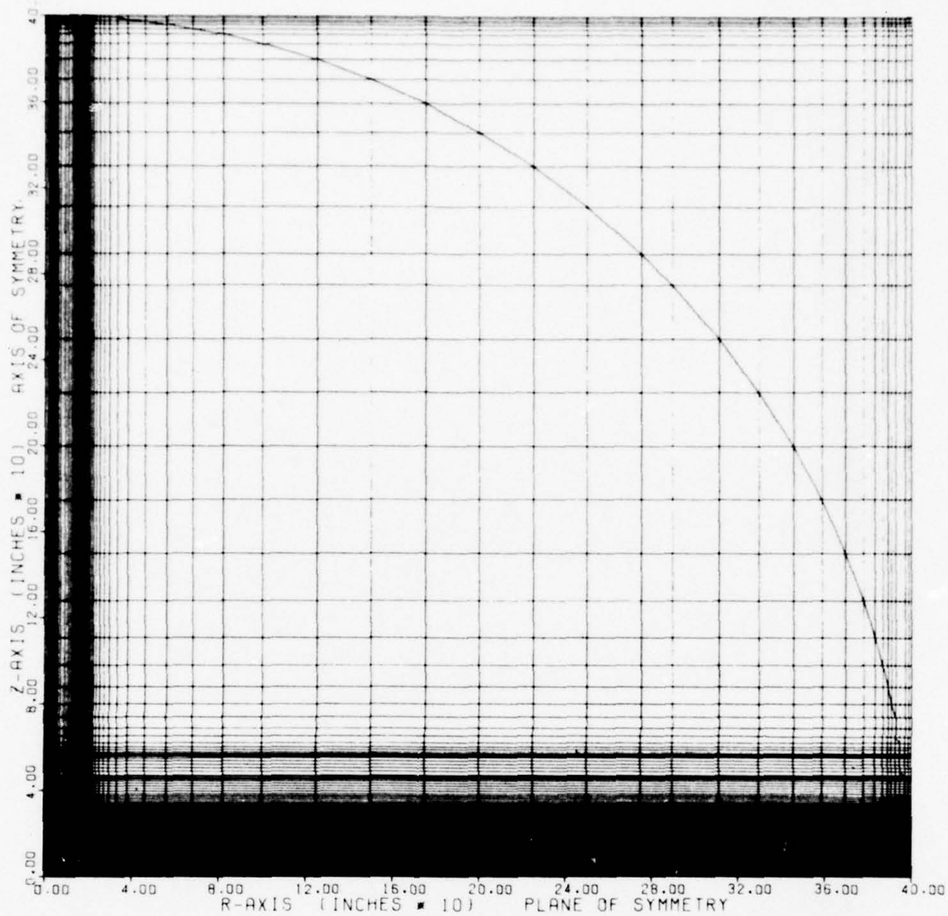


Figure 12. Coordinate grid for one quadrant of guy line insulator problem.

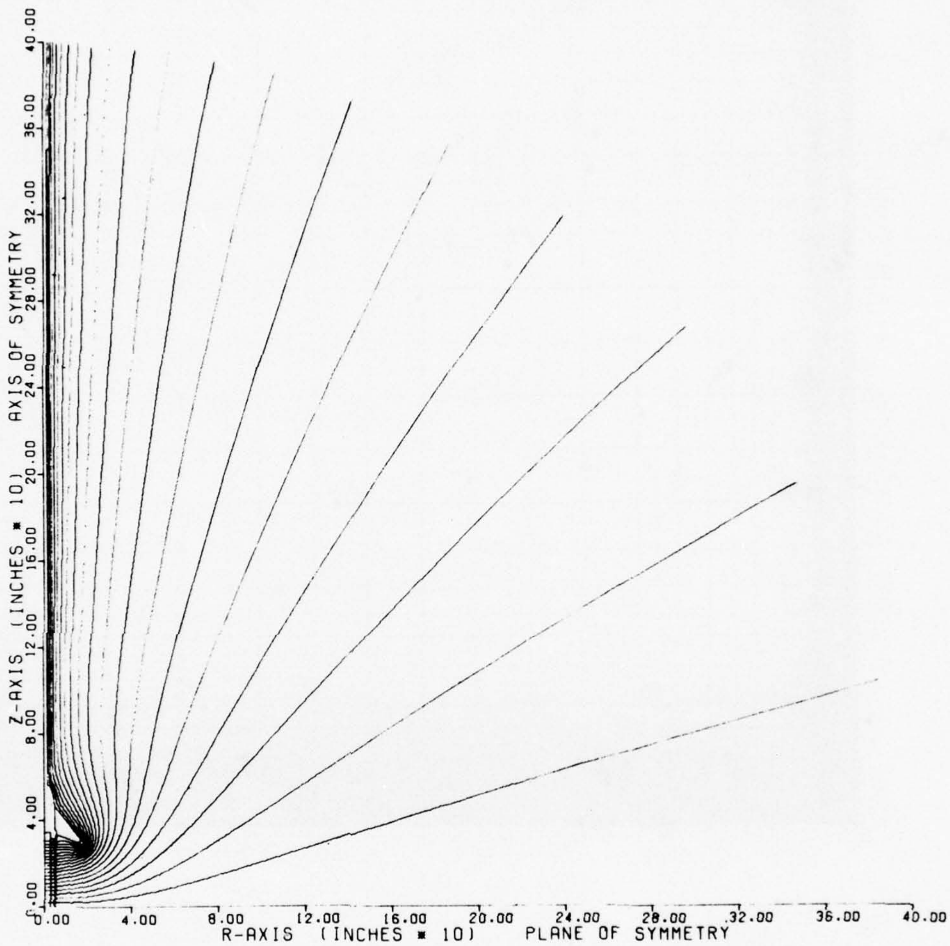


Figure 13. Equipotential plot for one quadrant of guy line insulator.

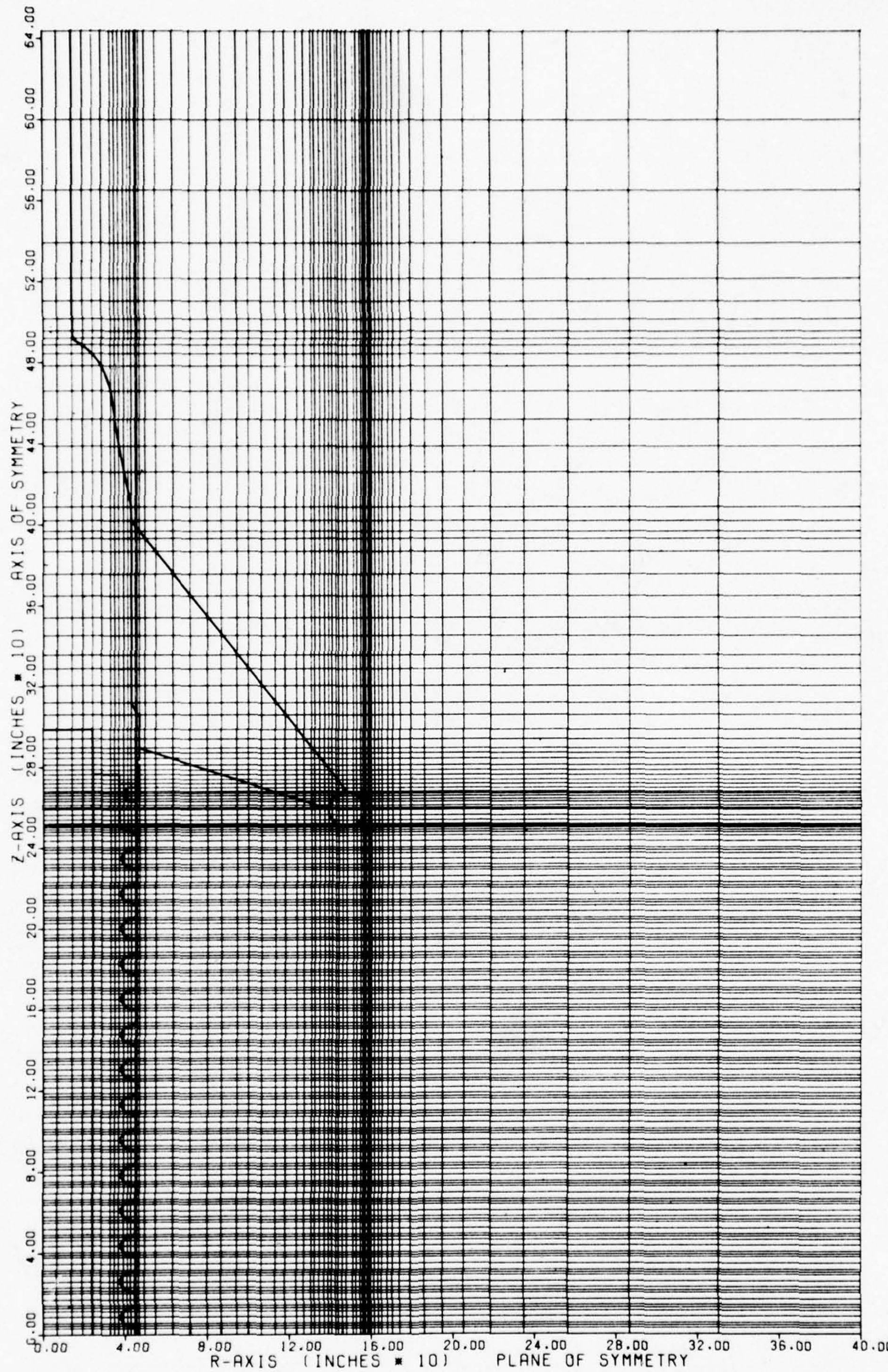


Figure 14. Enlarged view of coordinate grid in region of guy line insulator.

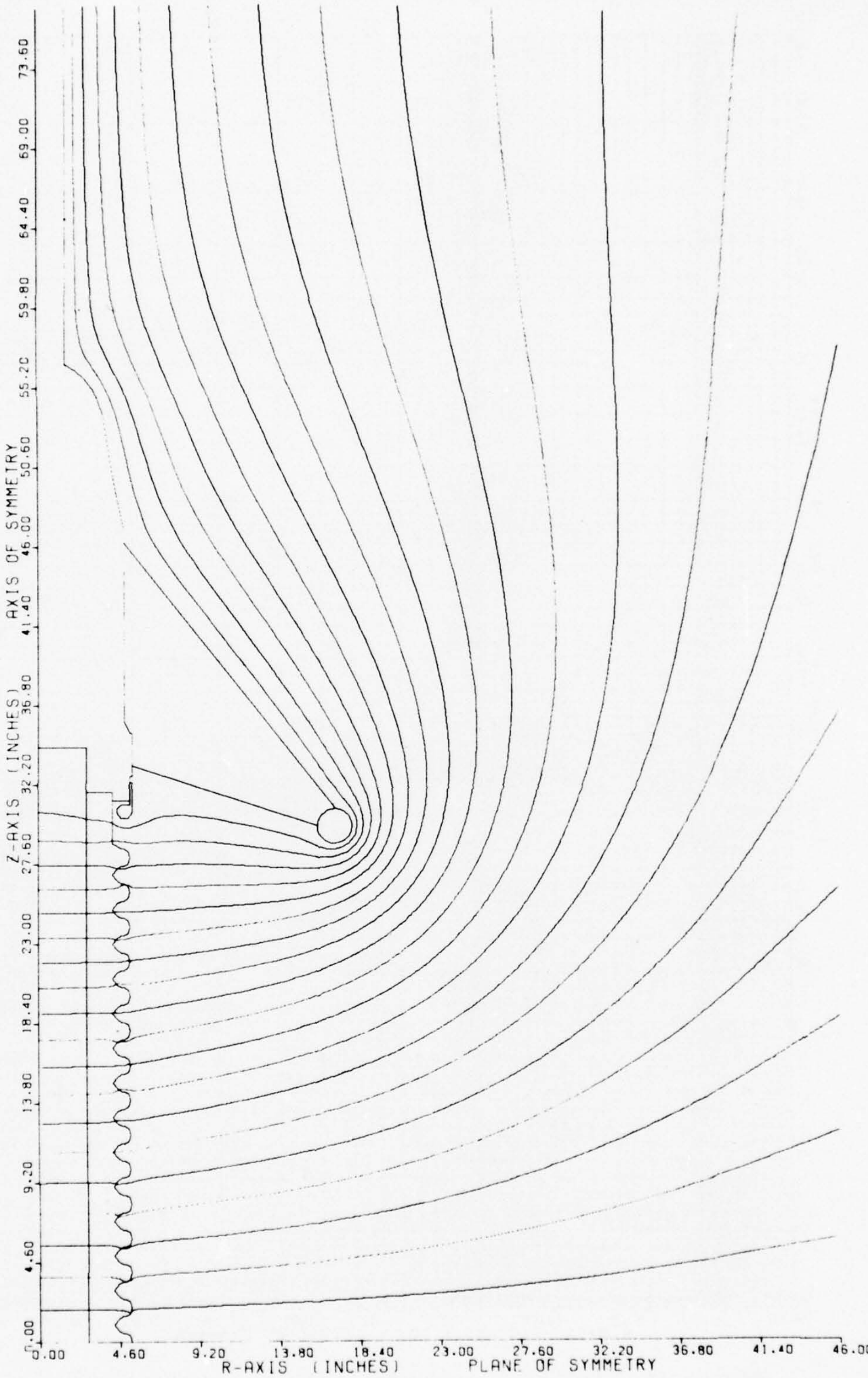


Figure 15. Equipotential plot of enlarged region of guy line insulator.

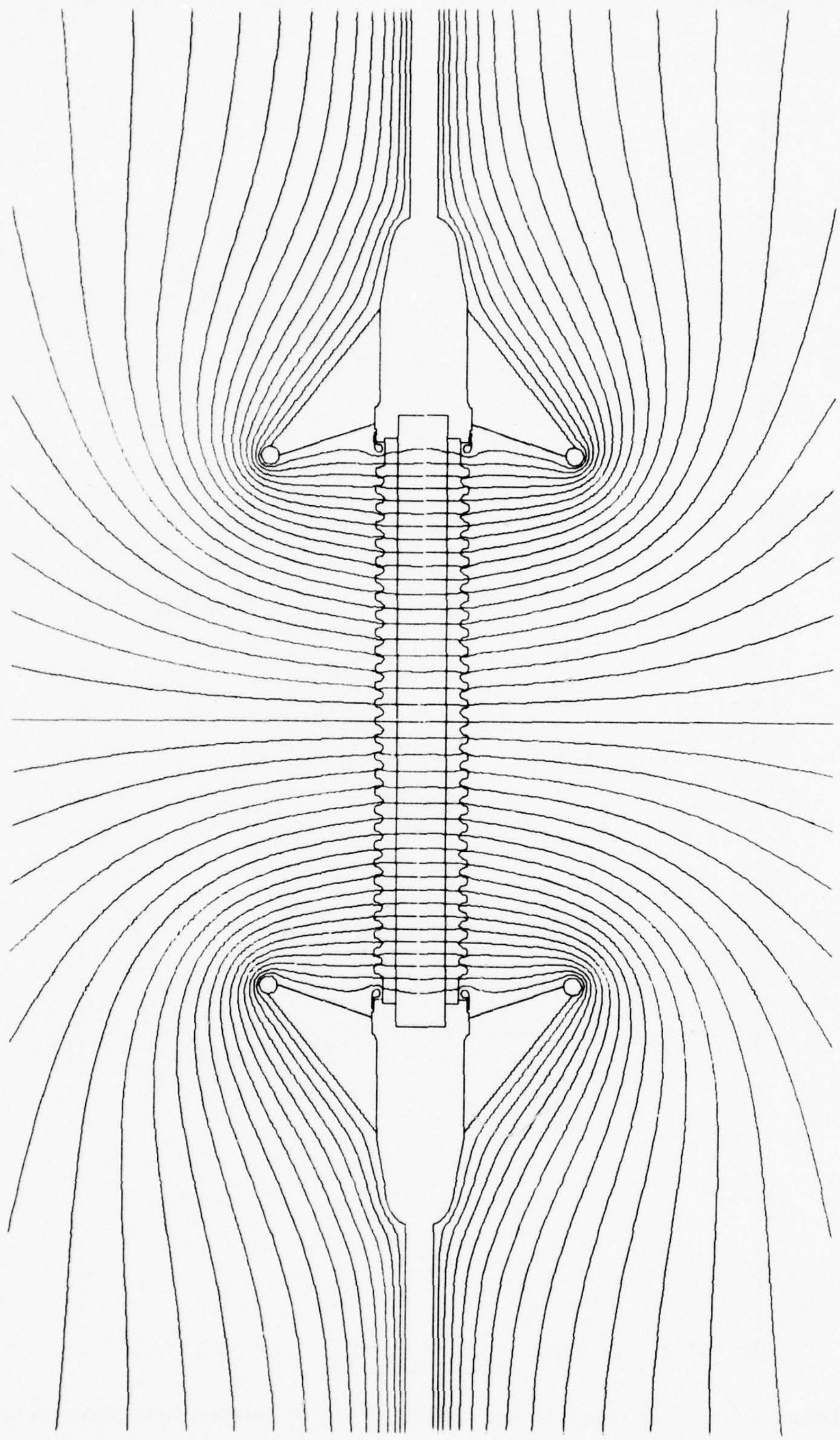


Figure 16. Equipotential plot of full view of guy line insulator.

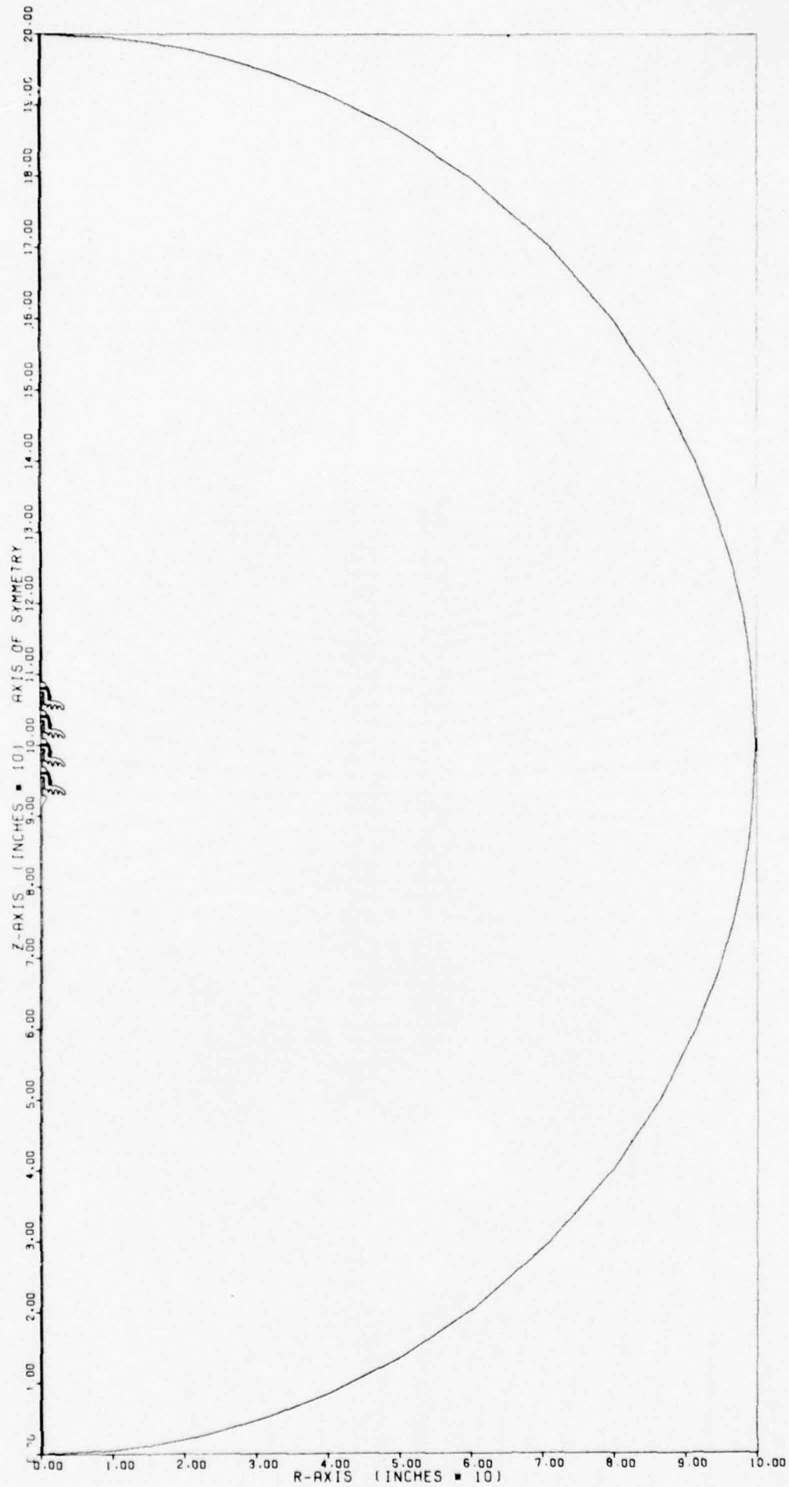


Figure 17. Grid region for chain of four suspension insulators.

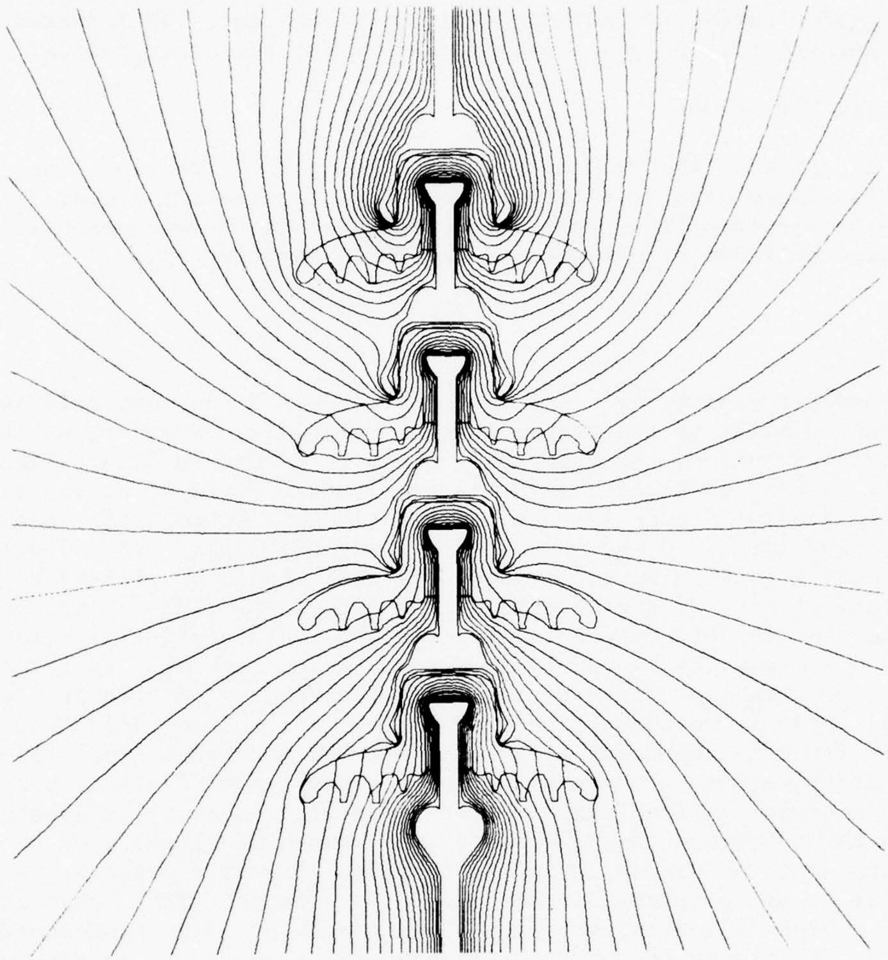


Figure 18. Enlarged full-section equipotential plot for chain of four suspension insulators.

Figure 18 is an enlarged, full-section view, equipotential plot of the suspension insulator chain. The voltage between each pair of adjacent equipotential lines is 2% of the total voltage across the end electrodes. The potentials of the two end electrodes may be arbitrarily specified, but the potentials of the interior electrodes are unknown prior to execution of the program. These conductors must be treated as dielectrics having an infinite dielectric constant. Their potentials are then calculated in the same manner as for all other points.

Electric Flux Lines

In addition to the equipotential lines, the direction lines of the electric field also give information about an electric field. These lines of electric flux, which are orthogonal to the equipotential lines, can also be found by solution of Laplace's Equation 35.

$$\nabla^2 U = 0 \tag{35}$$

The computer program will calculate a value of U for each grid point in a manner similar to that for electric potentials. However, unlike the potential solution, the numerical values of U have no direct significance. A line of equal U value is an electric field direction line, but, in general, there is no clear relationship between the spacing of these flux lines and the values of U along the lines. It is somewhat difficult to determine the values of U that should be plotted to give an accurate picture of the electric field intensity. This is in contrast to the plotting of equipotential lines, where the potential values to be plotted are equally spaced values of V. Also, care must be taken when using the flux line plots to determine the regions of highest electric field. This is because the electric flux is a volume field flow which is difficult to represent clearly in a two-dimensional plot. Electric flux distribution is shown by describing many tubes of flux, each of which encloses a given amout of flux. The flux density is greatest where the tubes are the narrowest. In a two-dimensional plot, a tube may appear to be narrow, when, in fact, the dimension that is perpendicular to the plotting surface may be very large. Thus, what appears to be a high flux density region may actually be a low flux density region. Knowledge of the three-dimensional geometry is necessary in order to interpret a two-dimensional plot.

In spite of the difficulty in interpretation, plots of electric flux lines are sometimes useful. They indicate the direction of the electric field more clearly than equipotential plots and can also be used with potential plots to more clearly define regions of high electric field intensity or distortion.

To obtain the solution to Equation 35 for the purpose of plotting flux lines, the computer program is executed exactly as for the potential solution. All that needs to be changed are the boundary conditions

and dielectric constants such that the equal U-value lines are orthogonal to the equipotential lines. The potential function, $V(r,z)$, has a gradient that is perpendicular to lines of equal V at each point. The negative of the gradient of V is the electric field vector function, $\bar{E}$, as in Equation 36.

$$\bar{E} = -\nabla V \tag{36}$$

For the function $U(r,z)$ there can be defined a corresponding vector function, $\bar{F}$, given by Equation 37.

$$\bar{F} = -\nabla U \tag{37}$$

$\bar{F}$ is perpendicular to the lines of equal U at each point. The requirement that lines of equal V be perpendicular to lines of equal U is equivalent to requiring that vector function $\bar{E}$ be perpendicular to vector function $\bar{F}$.

The requirement of having E perpendicular to F can be analyzed at a boundary between two dielectrics as shown in Figure 19. In (a) the dielectric constants ε_1 and ε_2 are normal dielectric constants as they would be used in the solution for the potential distribution, V. In (b) the dielectric constants, ε_1' and ε_2', are the values as they would be used to obtain the U distribution. $\hat{n}$ and $\hat{t}$ are, respectively, unit normal and unit tangent vectors to the boundary between the two media.

Since there is no current flowing along the boundary between the dielectrics, the tangential electric fields on each side of the boundary must be equal. This condition is expressed mathematically in Equation 38.

$$\hat{t}\bullet\bar{E}_1 = \hat{t}\bullet\bar{E}_2 \tag{38}$$

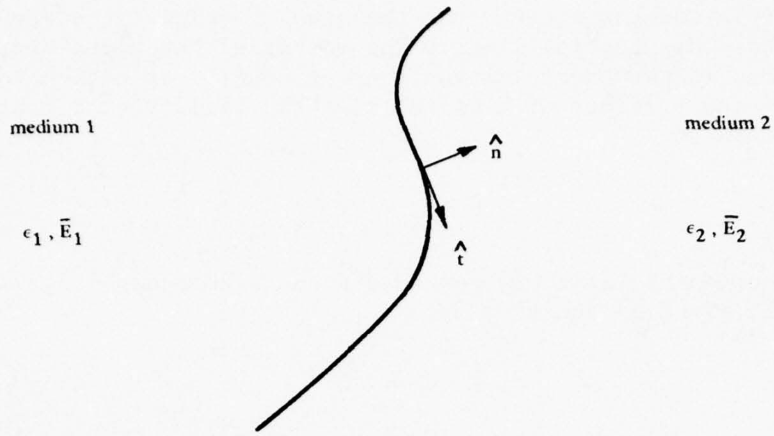
Also, the normal component of the displacement flux density must be continuous across the dielectric boundary as expressed in Equation 39.

$$\varepsilon_1(\hat{n}\bullet\bar{E}_1) = \varepsilon_2(\hat{n}\bullet\bar{E}_2) \tag{39}$$

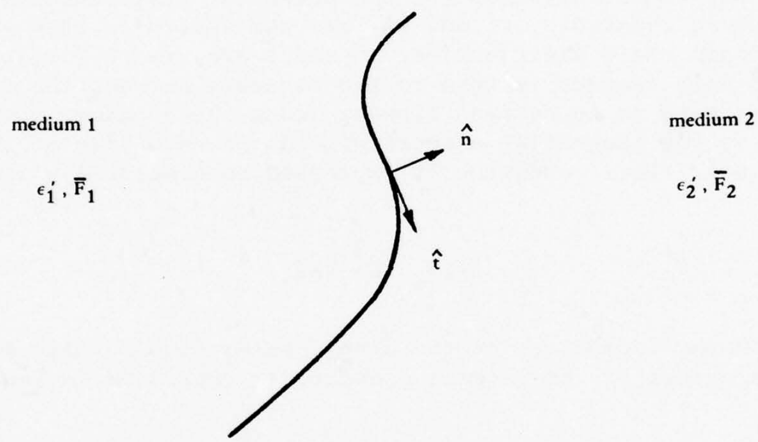
Further, $\bar{F}$ corresponds to $\bar{E}$, and both are actually solutions to the same equations with only the boundary conditions differing. Therefore, $\bar{F}$ must also meet conditions that correspond to Equations 38 and 39 for $\bar{E}$. These conditions for $\bar{F}$ are expressed in Equations 40 and 41.

$$\hat{t}\bullet\bar{F}_1 = \hat{t}\bullet\bar{F}_2 \tag{40}$$

$$\varepsilon_1'(\hat{n}\bullet\bar{F}_1) = \varepsilon_2'(\hat{n}\bullet\bar{F}_2) \tag{41}$$



(a) V problem.



(b) U problem.

Figure 19. Boundary between two dielectric media.

$\bar{E}$ and $\bar{F}$ can be expressed in terms of their tangential and normal components at the boundary between the two dielectrics. In medium 1 at the boundary:

$$\bar{E}_1 = \hat{t}\bullet\bar{E}_1 + \hat{n}\bullet\bar{E}_1 \tag{42}$$

$$\bar{F}_1 = \hat{t}\bullet\bar{F}_1 + \hat{n}\bullet\bar{F}_1 \tag{43}$$

In medium 2 at the boundary:

$$\bar{E}_2 = \hat{t}\bullet\bar{E}_2 + \hat{n}\bullet\bar{E}_2 \tag{44}$$

$$\bar{F}_2 = \hat{t}\bullet\bar{F}_2 + \hat{n}\bullet\bar{F}_2 \tag{45}$$

The requirement that $\bar{E}$ and $\bar{F}$ be perpendicular must hold at the dielectric boundary. The nonzero vectors $\bar{E}$ and $\bar{F}$ are perpendicular if and only if their dot product is zero as in Equation 46.

$$\bar{E}\bullet\bar{F} = 0 \tag{46}$$

Substituting Equations 42 and 43 into Equation 46 gives the condition for perpendicularity at the boundary in medium 1:

$$\bar{E}_1\bullet\bar{F}_1 = (\hat{t}\bullet\bar{E}_1)(\hat{t}\bullet\bar{F}_1) + (\hat{n}\bullet\bar{E}_1)(\hat{n}\bullet\bar{F}_1) = 0 \tag{47}$$

The condition for medium 2 is obtained by substituting Equations 44 and 45 into Equation 46 to yield:

$$\bar{E}_2\bullet\bar{F}_2 = (\hat{t}\bullet\bar{E}_2)(\hat{t}\bullet\bar{F}_2) + (\hat{n}\bullet\bar{E}_2)(\hat{n}\bullet\bar{F}_2) = 0 \tag{48}$$

Equations 47 and 48 are equivalent and can be compared by substitution of Equations 38, 39, 40, and 41 into Equation 47. Equation 49 is Equation 47 written in terms of $\bar{E}_2$ and $\bar{F}_2$.

$$(\hat{t}\bullet\bar{E}_2)(\hat{t}\bullet\bar{F}_2) + \frac{\varepsilon_2}{\varepsilon_1}(\hat{n}\bullet\bar{E}_2)\frac{\varepsilon_2'}{\varepsilon_1'}(\hat{n}\bullet\bar{F}_2) = 0 \tag{49}$$

Equations 48 and 49 can both be true only if the dielectric constants satisfy Equation 50.

$$\frac{\varepsilon_2 \varepsilon_2'}{\varepsilon_1 \varepsilon_1'} = 1 \tag{50}$$

Equation 50 holds only if, for any value of α,

$$\varepsilon_2' = \frac{\alpha}{\varepsilon_2} \tag{51}$$

$$\varepsilon_1' = \frac{\alpha}{\varepsilon_1} \tag{52}$$

For simplicity, α can be chosen to be unity. Equations 53 and 54 give the dielectric constant changes used when it is desired to obtain the U solution for plotting electric flux lines that are perpendicular to electric potential lines for a given dielectric configuration.

$$\varepsilon_1' = \frac{1}{\varepsilon_1} \tag{53}$$

$$\varepsilon_2' = \frac{1}{\varepsilon_2} \tag{54}$$

In addition to changing the dielectric constants, the boundary conditions must also be chosen so as to give a solution of electric flux lines that are perpendicular to equipotential lines. If a grid boundary corresponds to what is known to be a flux line, that portion of the grid boundary may be assigned a constant value, U_1. This condition often applies to the section of the axis of symmetry between the electrodes of an insulator. For boundaries that are distant from the insulator assembly, a curve that approximates a known electric flux line can be estimated as described earlier for the equipotential solution. The points along this curve may be assigned a second constant value, U_2. Some portions of the grid boundary may correspond to an equipotential line or an electrode or other conductor. The electric flux lines must intersect these boundaries in a perpendicular manner. This condition is met by describing the region beyond the equipotential line or electrode boundary as having a dielectric constant of zero. The argument here is consistent with that for the potential distribution problem where an electric flux line and zero dielectric constant are used to satisfy certain boundary conditions.

Figure 20 shows the boundary conditions for a potential distribution problem. The corresponding boundary conditions for obtaining the electric flux line solution are shown in Figure 21. The geometry of the problem and the dielectrics involved are designed to give a simple illustration of the concept and represent no actual device.

The enlarged equipotential plot is given in Figure 22 while both the equipotential lines and the lines of displacement flux density are shown in Figure 23. They must be referred to as lines of displacement flux density, $\bar{D}$, since their density is continuous across the dielectric boundaries. It is more practical to plot the flux as lines of $\bar{D}$ rather than as lines of $\bar{E}$, since $\bar{E}$ is not continuous across the boundaries between dielectrics. If lines of $\bar{E}$ were plotted, they would need to have a density in the dielectrics that is inversely proportional to each dielectric constant. Thus, there would be fewer lines in the higher dielectrics than are shown in Figure 23.

In order to interpret the displacement flux density plot, it is important to understand the meaning of the line spacing. In Figure 23 the line spacing is determined according to the displacement flux density which flows in the plane of the figure. Therefore, it gives only a two-dimensional representation of the flux distribution. The third dimension must be visualized by rotation of Figure 23 about the axis of symmetry. The significance of this rotation is that the lines nearer the axis sweep out a smaller volume than do the lines that are far from the axis of symmetry. A given amount of flux in a smaller volume has a higher flux density. Therefore, the regions near the axis of symmetry have a higher relative flux density than appears in Figure 23.

DISCUSSION

The computer program is designed to yield a plot of equipotential lines. From the equipotential plot it is easy to visually determine the regions of highest electric field. These regions are characterized by closely packed equipotential lines. In the equipotential plot there is a fixed potential difference between any two adjacent equipotential lines. A relatively short distance between lines indicates a high voltage gradient. As discussed earlier, the voltage gradient is the dominant component of the electric field near the insulator and electrodes at VLF and lower frequencies. The regions having highest voltage gradients are, therefore, most susceptible to electrical breakdown.

Since the equations used in the development of the computer program are valid for all frequencies up to and including VLF, the program has a wide range of application. Most notably, it is a powerful tool that can be used in the analysis of most 60-Hertz power system insulators.

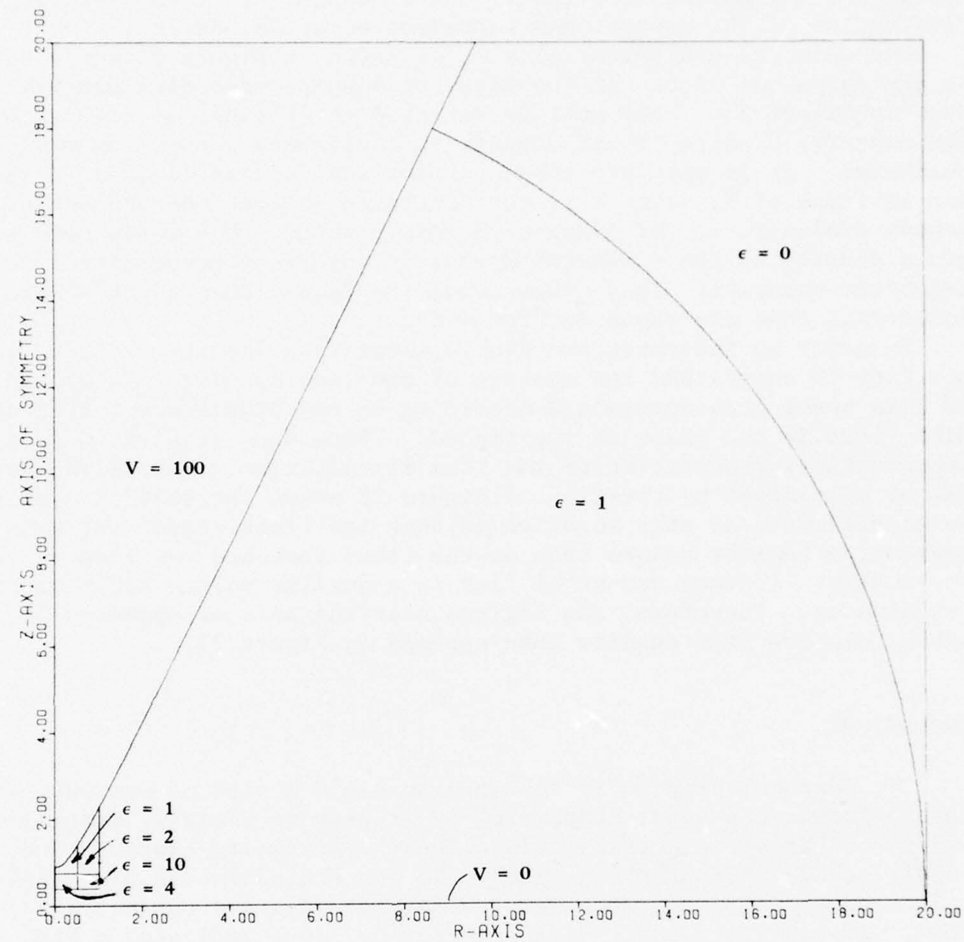


Figure 20. Boundary conditions for potential distribution problem.

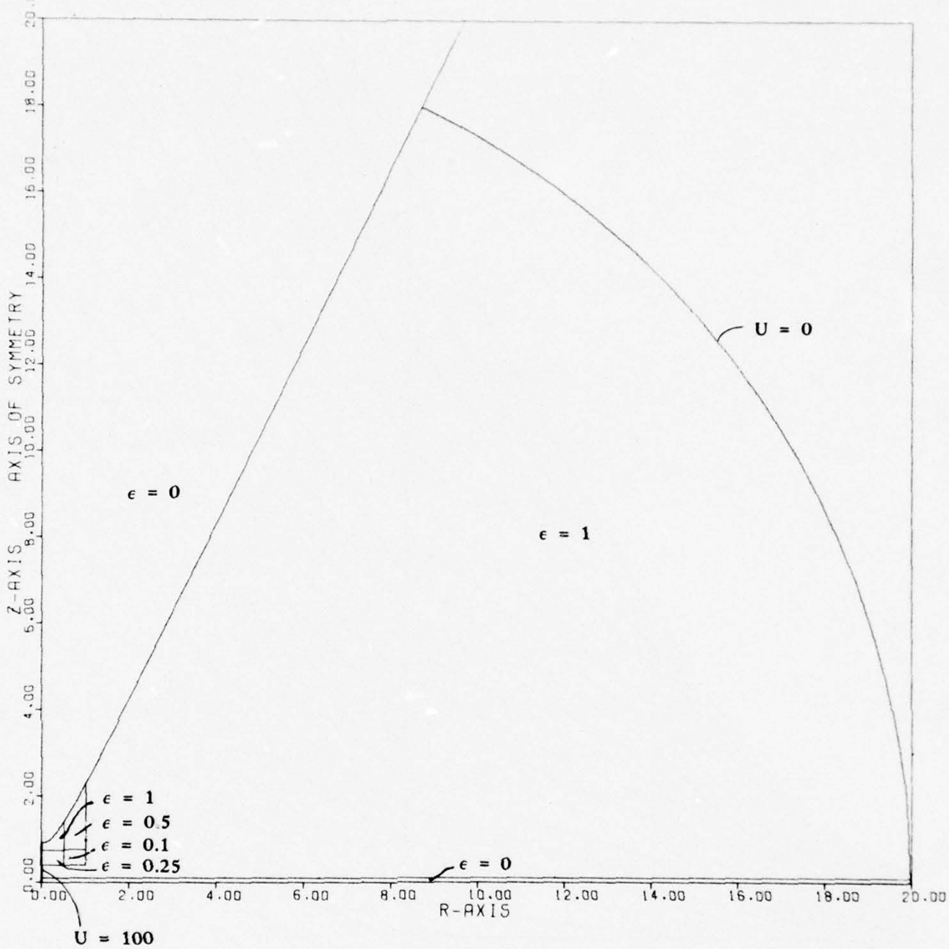


Figure 21. Boundary conditions for electric flux distribution problem.

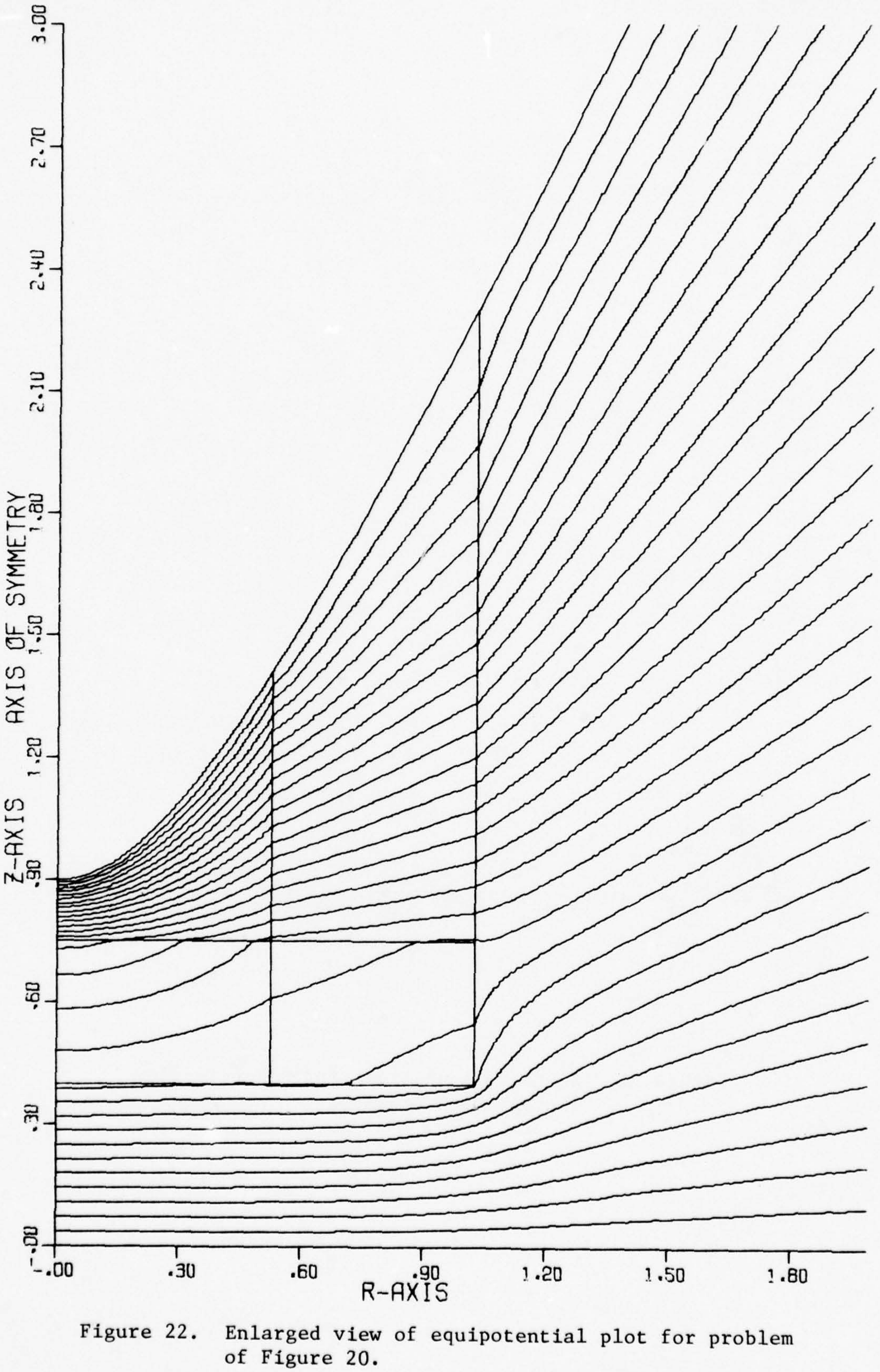


Figure 22. Enlarged view of equipotential plot for problem of Figure 20.

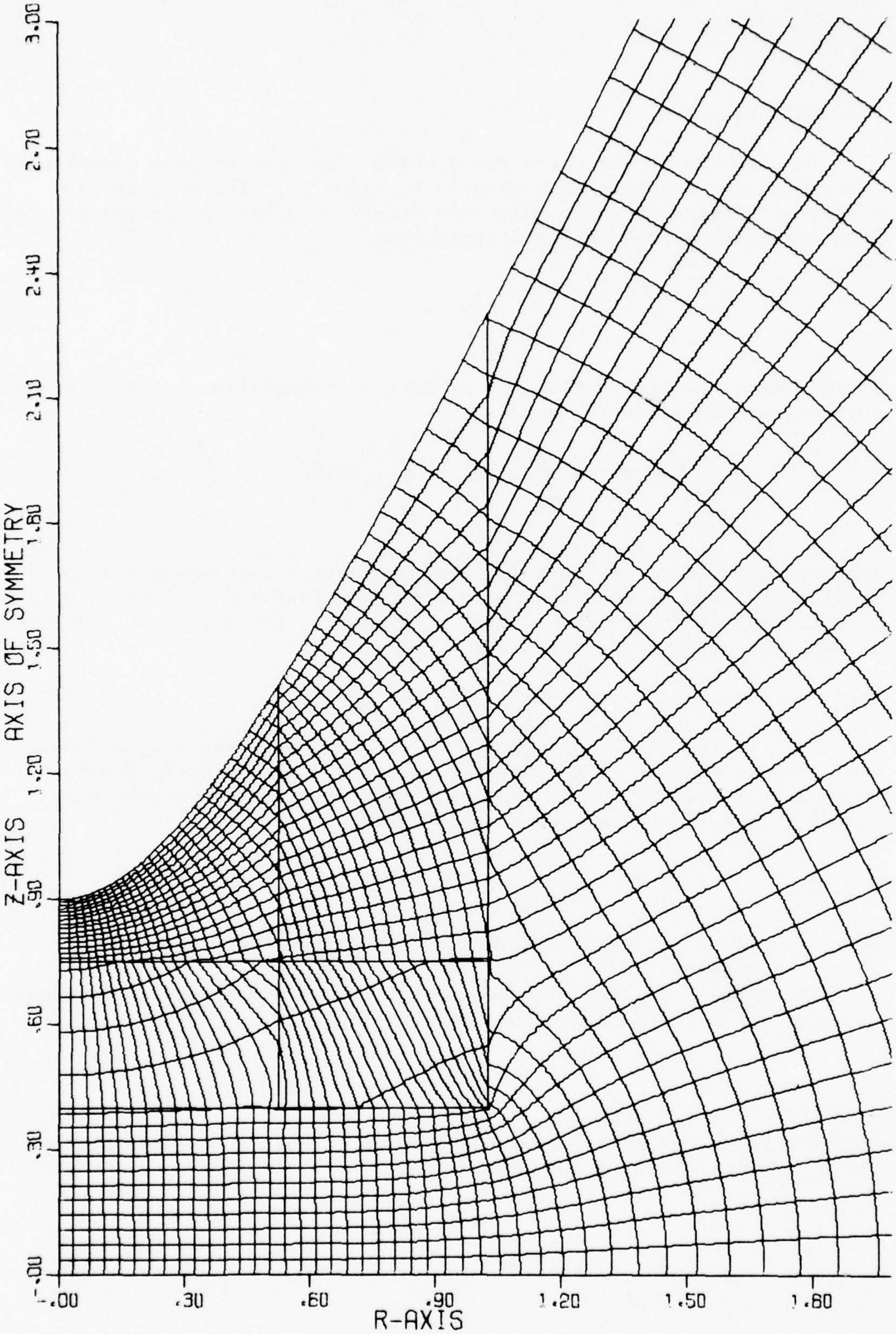


Figure 23. Enlarged view of equipotential and flux line plot for problem of Figures 20 and 21.

Poisson's Equation

In determining the potential distribution, the program essentially calculates the solution to Laplace's Equation 20. The program can easily be adapted to solve Poisson's Equation 55, which applies to problems involving a free charge distribution.

$$\nabla^2 V = \frac{\rho}{\varepsilon} \tag{55}$$

In this case the charge density, ρ, must be integrated over each volume element, and Equation 29 becomes:

$$\int_{vol} \bar{\nabla}\cdot\bar{D}\, dv = \oint_{s} \bar{D}\cdot d\bar{s} = \oint_{s} \varepsilon\, \nabla V\cdot d\bar{s} = \int_{vol} \rho\, dv \tag{56}$$

This has the effect of replacing the zero on the right-hand side of Equations 31 and 34 with a term that represents the net charge distributed throughout the volume element. Otherwise, the method of solution is the same as described earlier.

Other Applications

The program is suitable for solving many other types of problems that involve Laplace's or Poisson's equation. For example, Equation 57 is Laplace's equation as it applies to steady-state conduction heat transfer in regions where there is no heat generation.

$$\nabla^2 u = 0 \tag{57}$$

Here, u represents the heat energy or temperature as a function of position in space.

In problems dealing with mechanical stress there sometimes arises an equation of the form of Equation 58.

$$\nabla^4 A = 0 \tag{58}$$

This can be solved by the computer program through a two-step process if there can be found an X such that

$$\nabla^2 A = X \tag{59}$$

If the boundary conditions for X are known, the complete solution for X can be found by solving

$$\nabla^2 X = 0 \tag{60}$$

Equation 60, which is Laplace's equation, is identical to Equation 58 with the substitution for A made as in Equation 59. Then A itself can be found by solution of Equation 59, which is Poisson's equation, utilizing the boundary conditions for A and the solution for X.

Two-Dimensional Geometries

If the geometry of a problem has only two dimensions, the equations are much simpler than for three dimensions with one axis of symmetry. The volume element of Figure 4 is reduced to a surface element as shown in Figure 24 for a rectangular coordinate system. Instead of summing the flux crossing surface areas, the sum of the flux which crosses the boundaries of the surface in Figure 24 is taken. Equation 61 is the new equation which corresponds to Equation 31.

$$\begin{aligned}
&\frac{\phi_1}{h_1}(\varepsilon_1 h_2 + \varepsilon_8 h_4) + \frac{\phi_2}{h_2}(\varepsilon_2 h_1 + \varepsilon_3 h_3) \\
&+ \frac{\phi_3}{h_3}(\varepsilon_4 h_2 + \varepsilon_5 h_4) + \frac{\phi_4}{h_4}(\varepsilon_6 h_3 + \varepsilon_7 h_1) \\
&- \phi_o \left[\frac{1}{h_1}(\varepsilon_1 h_2 + \varepsilon_8 h_4) + \frac{1}{h_2}(\varepsilon_2 h_1 + \varepsilon_3 h_3)\right. \\
&\left. + \frac{1}{h_3}(\varepsilon_4 h_2 + \varepsilon_5 h_4) + \frac{1}{h_4}(\varepsilon_6 h_3 + \varepsilon_7 h_1)\right] = 0
\end{aligned} \tag{61}$$

Equation 61 is a numerical form of Equation 29 where the surfaces have been reduced to straight-line boundaries, and a factor of one-half has been removed from each term.

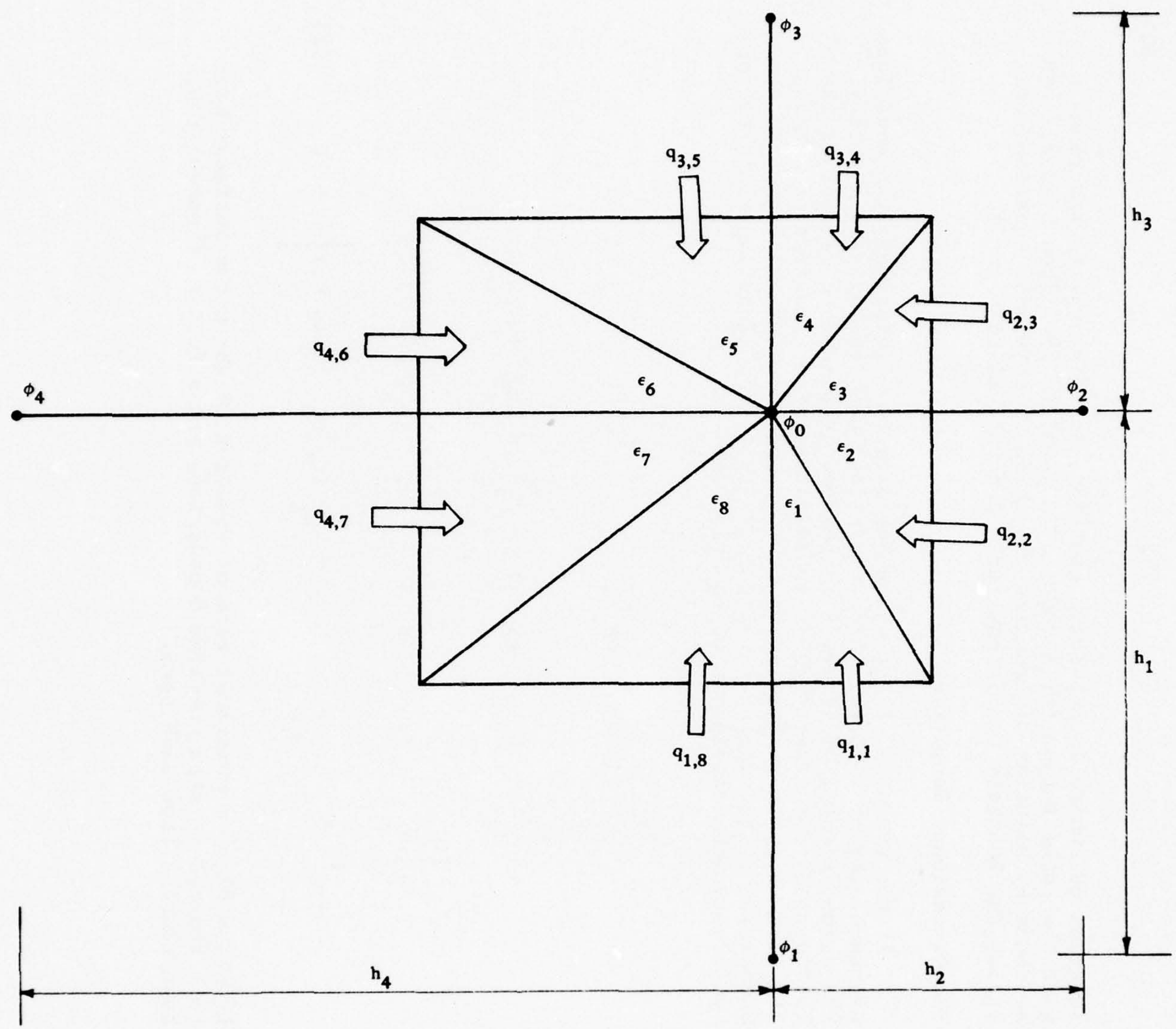


Figure 24. Surface element for two-dimensional problem.

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Appendix

FVSOLVR and FVPLOT

The computer listing that follows is given in the form of two main programs. The first, FVSOLVR, reads input data, formulates, and solves the equations, giving the values of potentials at each grid intersection point. The second program, FVPLOT, uses the potential distribution solution from FVSOLVR to plot lines of equal potential. It is convenient to separate the process into two programs so that the plotting is detached from the solution calculation. Thus, it is easy for the user to obtain several different plots for a given problem by specifying various equipotential value spacings or various regions of the grid to be plotted. FVPLOT can plot equal value lines from up to two sets of data, such as equipotential lines and flux lines, on one grid. If both equipotential and flux lines are to be plotted, then two solutions from FVSOLVR are required before FVPLOT is used.

FVPLOT is designed to produce Calcomp pen plots on drum plotters having a width of 11 inches. In the form listed, FVPLOT utilizes the CDC 7600 computer at the Lawrence Berkeley Laboratory via remote terminal access.

Comment statements given throughout the listings indicate the general organization of the programs. A detailed users manual is in preparation.

```
C
C     INSULATOR FIELD VALUE SOLVER (AXIAL SYMMETRY)
C
      DIMENSION HX(100),HY(200),HN(99),HM(200),AN(99),
     1EPSR(199,10),EPSZ(99,10),ANEPS(99),RHN(99),SYM(200,4),
     2ZEPS1(99),V(100,200),B(200),AR(200,100)
      DIMENSION GH(200),VG(200)
      INTEGER BDR(199,10),BDZ(99,10)
      INTEGER BLNK
      COMMON/BANARG/NDPN,NUMBLK,B,AR,NN,NN2,V,NX,NY,LX,LY,M,NCK8
      COMMON/HORARG/BDR,EPSR
      COMMON/VERARG/BDZ,EPSZ
      COMMON/SPARG/GH,VG,HM
      DATA BLNK/4H    /
      DATA AR/20000*0.0/,SYM/800*0.0/,B/200*0.0/,V/20000*-1.0/
C
C     FORMAT STATEMENTS
C
    1 FORMAT(10F8.0)
    2 FORMAT(20I4)
    3 FORMAT(16F5.0)
    4 FORMAT(8F10.3)
    5 FORMAT(1H1,4HNX =,I4,4X,4HNY =,I4)
    6 FORMAT(1H ,A1,3HHX(,I3,3H) =,F8.3,4(A4,6X,3HHX(,I3,3H) =,F8.3))
    7 FORMAT(1H ,A1,3HHY(,I3,3H) =,F8.3,4(A4,6X,3HHY(,I3,3H) =,F8.3))
    8 FORMAT(1H1,9X,1H+,14F8.3)
    9 FORMAT(10X,1H+,14F8.3)
   10 FORMAT(10X,1H+,14(8H----+---))
   11 FORMAT(1X,F8.3,2H +,14F8.3)
   12 FORMAT(1H0,15HINITIAL VALUE =,F8.3)
   13 FORMAT(1H0,6HNPTS =,I7)
   14 FORMAT(1X,7HNUMBLK=,I5)
   15 FORMAT(1X,5HNCK5=,I5)
   16 FORMAT(1X,6HNTEST=,I5)
   17 FORMAT(13H NONSYMMETRIC)
   18 FORMAT(1H1)
   19 FORMAT(1H )
   20 FORMAT(1H0)
   21 FORMAT(1H-)
   22 FORMAT (1H ,27HREADY TO BEGIN CALCULATIONS)
   23 FORMAT(2I8,4F12.3)
C
C     INITIALIZATION
C
      READ  2 ,NX,NY,NEPR,NEPZ,IFBDY
      PRINT  5 ,NX,NY
      PRINT  20
      NXM1=NX-1
      NYM1=NY-1
      NN=NX
      NN2=NN*2
      NNP1=NN+1
      REWIND 10
      REWIND 33
      REWIND 34
C
C     READ GRID DATA
```

```
C
      READ  1 ,(HX(I),I=1,NX)
      K1=0
      NCOL=NXM1/50
      GOTO 120
  100 NCOL=NCOL-1
  120 INK=NCOL*50
  130 K1=K1+1
      K2=K1+INK
      PRINT  6 ,(BLNK,K,HX(K),K=K1,K2,50)
      IF (K1.EQ.50.OR.K1.EQ.NX) GOTO 140
      IF (K2.EQ.NX) 100,130
  140 READ  1 ,(HY(J),J=1,NY)
      PRINT  18
      K1=0
      NCOL=NYM1/50
      GOTO 170
  150 NCOL=NCOL-1
  170 INK=NCOL*50
  180 K1=K1+1
      K2=K1+INK
      PRINT  7 ,(BLNK,K,HY(K),K=K1,K2,50)
      IF (K1.EQ.50.OR.K1.EQ.NY) GOTO 200
      IF (K2.EQ.NY) 150,180
  200 READ  2 ,((BDR(I,J),J=1,NEPR),I=1,NYM1)
  220 READ  2 ,KO,KN,KK
      READ  3 ,(EPSR(KO,K),K=1,KK)
      IF (KO.EQ.KN) GOTO 260
      KI=KO+1
      DO 240 I=KI,KN
      DO 240 K=1,KK
  240 EPSR(I,K)=EPSR(KO,K)
  260 IF (KK.EQ.NEPR) GOTO 300
      KP=KK+1
      DO 280 I=KO,KN
      DO 280 J=KP,NEPR
  280 EPSR(I,J)=EPSR(I,KK)
  300 IF (KN.LT.NYM1) GOTO 220
      READ  2 ,((BDZ(I,J),J=1,NEPZ),I=1,NXM1)
  320 READ  2 ,KO,KN,KK
      READ  3 ,(EPSZ(KO,K),K=1,KK)
      IF (KO.EQ.KN) GOTO 360
      KI=KO+1
      DO 340 I=KI,KN
      DO 340 K=1,KK
  340 EPSZ(I,K)=EPSZ(KO,K)
  360 IF (KK.EQ.NEPZ) GOTO 400
      KP=KK+1
      DO 380 I=KO,KN
      DO 380 J=KP,NEPZ
  380 EPSZ(I,J)=EPSZ(I,KK)
  400 IF (KN.LT.NXM1) GOTO 320
      IF (IFBDY.EQ.0) GOTO 550
C
C     INTERPOLATE FINE GRID BOUNDARY POTENTIALS
C     FROM COARSE GRID POTENTIAL SOLUTION
C
```

```
      READ  2 ,NGX,NGY
      READ  4 ,(GH(I),I=1,NGX)
      READ  4 ,(VG(I),I=1,NGX)
      CALL SPCOEF (NGX)
      DO 450 I=1,NX
  450 V(I,NY)=SPLINE(NGX,HX(I))
      READ  4 ,(GH(I),I=1,NGY)
      READ  4 ,(VG(I),I=1,NGY)
      CALL SPCOEF (NGY)
      DO 500 I=1,NYM1
  500 V(NX,I)=SPLINE(NGY,HY(I))
C
C     READ BOUNDARY CONDITIONS
C
  550 READ  2 ,NPOTS
      PRINT  18
      DO 600 N=1,NPOTS
      READ  1 ,POT
      PRINT  12,POT
      READ  2 ,NLINES
      PRINT  2 ,NLINES
      DO 600 K=1,NLINES
      READ  2 ,L1,L2,M1,M2
      PRINT 2 ,L1,L2,M1,M2
      DO 600 I=L1,L2
      DO 600 J=M1,M2
  600 V(I,J)=POT
C
C     PARAMETER INITIALIZATION
C
      NUMBLK=0
      NCK=0
      NCK1=1
      NCK4=NN
      NCK5=0
      NCNT=0
      LY=2
      Z=(HY(1)+HY(2))/2.
      ZPH=(HY(2)+HY(3))/2.
      NUMX=NX-1
      IF (HX(1).NE.0.0) NUMX=NX-2
      NPTS=NUMX*(NY-2)
      PRINT  20
      PRINT  13,NPTS
      NCK7=NPTS-NUMX
      NDIAG=1
      NDP1=NDIAG+1
      NDPN=NDIAG+NUMX
      PRINT  18
      DO 650 K=1,NXM1
  650 ZEPS1(K)=EPSZ(K,1)
C
C     CALCULATE DISTANCES BETWEEN GRID LINES
C
      DO 700 J=1,NXM1
  700 HN(J)=HX(J+1)-HX(J)
      DO 720 J=1,NYM1
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```
  720 HM(J)=HY(J+1)-HY(J)
C
C     CALCULATE AREAS OF HORIZONTAL FACES, SET RHN ARRAY
C
      R=HX(1)
      AN(1)=0.5*HN(1)*(R+HN(1)/4.)
      RHN(1)=R+HN(1)/2.
      DO 750 J=2,NXM1
      RHN(J)=(HX(J)+HX(J+1))/2.
  750 AN(J)=(RHN(J)**2-RHN(J-1)**2)/2.
C
C     BOTTOM EDGE AREA - EPS DATA SET-UP
C
      EPS1=EPSR(1,1)
      R=HX(1)
      LX=1
      J=1
      CALL HORIZ(1,J,EPS1,R,RHN(LX),ANEPS(LX),AN(LX),LX,HX(LX))
      DO 800 LX=2,NXM1
      J=1
      CALL HORIZ(1,J,EPS1,RHN(LX-1),RHN(LX),ANEPS(LX),AN(LX),LX,HX(LX))
  800 CONTINUE
      PRINT  22
      PRINT  20
      GOTO 3000
C
C     SHIFT BLOCK UP AND ZERO
C
  900 DO 1000 K=1,NN
      KPNN=K+NN
      DO 950 I=1,4
      SYM(K,I)=SYM(KPNN,I)
  950 SYM(KPNN,I)=0.0
      B(KPNN)=0.0
      DO 1000 I=1,NN
 1000 AR(KPNN,I)=0.0
C
C     CALCULATE BLOCK OF AR, B ARRAY VALUES
C
 3000 DO 3600 L=NNP1,NN2
      IF (NCK.EQ.0) GOTO 3010
      LX=LX+1
      R=HX(LX)
      IF (LX.LT.NX) GOTO 3400
      LY=LY+1
      IF (LY.EQ.NY) GOTO 4000
      Z=ZPH
      ZPH=HY(LY)+HM(LY)/2.
 3010 J=1
      NCK=NCK+1
      EPS1=EPSR(LY,1)
      IF (HX(1).NE.0.0) GOTO 3200
      LX=1
      R=0.0
      IF (V(LX,LY).GE.0.0) GOTO 3300
      SYM(L,1)=0.0
      SYM(L,3)=ANEPS(1)/HM(LY-1)
```

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```
      CALL HORIZ(LY,J,EPS1,R,RHN(LX),ANEPS(LX),AN(LX),LX,HX(LX))
      SYM(L,4)=ANEPS(1)/HM(LY)
      CALL VERT(LX,ZEPS1(LX),Z,ZPH,RHN(LX),AZEPS,LY,HY(LY))
      SYM(L,2)=AZEPS/HN(LX)
      AR(L,NDIAG)=-SYM(L,1)-SYM(L,2)-SYM(L,3)-SYM(L,4)
      IF (V(LX,LY-1).LT.0.0) GOTO 3170
      B(L)=-V(LX,LY-1)*SYM(L,3)
      SYM(L,3)=0.0
3170  IF (V(LX+1,LY).LT.0.0) GOTO 3180
      B(L)=B(L)-V(LX+1,LY)*SYM(L,2)
      SYM(L,2)=0.0
3180  IF (V(LX,LY+1).LT.0.0) GOTO 3190
      B(L)=B(L)-V(LX,LY+1)*SYM(L,4)
      SYM(L,4)=0.0
3190  AR(L,NDP1)=SYM(L,2)
      AR(L,NDPN)=SYM(L,4)
      GOTO 3600
3200  LX=2
      R=HX(LX)
      IF (V(LX,LY).GE.0.0) GOTO 3330
      SYM(L,3)=ANEPS(2)/HM(LY-1)
      CALL HORIZ(LY,J,EPS1,RHN(LX-1),RHN(LX),ANEPS(LX),AN(LX),LX,HX(LX))
      SYM(L,4)=ANEPS(2)/HM(LY)
      CALL VERT(LX-1,ZEPS1(LX-1),Z,ZPH,RHN(LX-1),AZEPS,LY,HY(LY))
      SYM(L,1)=AZEPS/HN(LX-1)
      CALL VERT(LX,ZEPS1(LX),Z,ZPH,RHN(LX),AZEPS,LY,HY(LY))
      SYM(L,2)=AZEPS/HN(LX)
      AR(L,NDIAG)=-SYM(L,1)-SYM(L,2)-SYM(L,3)-SYM(L,4)
      B(L)=-V(1,LY)*SYM(L,1)
      SYM(L,1)=0.0
      IF (V(LX,LY-1).LT.0.0) GOTO 3270
      B(L)=B(L)-V(LX,LY-1)*SYM(L,3)
      SYM(L,3)=0.0
3270  IF (V(LX+1,LY).LT.0.0) GOTO 3280
      B(L)=B(L)-V(LX+1,LY)*SYM(L,2)
      SYM(L,2)=0.0
3280  IF (V(LX,LY+1).LT.0.0) GOTO 3290
      B(L)=B(L)-V(LX,LY+1)*SYM(L,4)
      SYM(L,4)=0.0
3290  AR(L,NDP1)=SYM(L,2)
      AR(L,NDPN)=SYM(L,4)
      GOTO 3600
3300  AR(L,NDIAG)=1.
      B(L)=V(LX,LY)
      CALL HORIZ(LY,J,EPS1,R,RHN(LX),ANEPS(LX),AN(LX),LX,HX(LX))
      CALL VERT(LX,ZEPS1(LX),Z,ZPH,RHN(LX),AZEPS,LY,HY(LY))
      GOTO 3600
3330  AR(L,NDIAG)=1.
      B(L)=V(LX,LY)
      CALL HORIZ(LY,J,EPS1,RHN(LX-1),RHN(LX),ANEPS(LX),AN(LX),LX,HX(LX))
      CALL VERT(LX,ZEPS1(LX),Z,ZPH,RHN(LX),AZEPS,LY,HY(LY))
      GOTO 3600
3400  IF (V(LX,LY).GE.0.0) GOTO 3330
      SYM(L,3)=ANEPS(LX)/HM(LY-1)
      CALL HORIZ(LY,J,EPS1,RHN(LX-1),RHN(LX),ANEPS(LX),AN(LX),LX,HX(LX))
      SYM(L,4)=ANEPS(LX)/HM(LY)
      SYM(L,1)=AZEPS/HN(LX-1)
```

```
      CALL VERT(LX,ZEPS1(LX),Z,ZPH,RHN(LX),AZEPS,LY,HY(LY))
      SYM(L,2)=AZEPS/HN(LX)
      AR(L,NDIAG)=-SYM(L,1)-SYM(L,2)-SYM(L,3)-SYM(L,4)
      IF (V(LX-1,LY).LT.0.0) GOTO 3460
      B(L)=-V(LX-1,LY)*SYM(L,1)
      SYM(L,1)=0.0
 3460 IF (V(LX,LY-1).LT.0.0) GOTO 3470
      B(L)=B(L)-V(LX,LY-1)*SYM(L,3)
      SYM(L,3)=0.0
 3470 IF (V(LX+1,LY).LT.0.0) GOTO 3480
      B(L)=B(L)-V(LX+1,LY)*SYM(L,2)
      SYM(L,2)=0.0
 3480 IF (V(LX,LY+1).LT.0.0) GOTO 3490
      B(L)=B(L)-V(LX,LY+1)*SYM(L,4)
      SYM(L,4)=0.0
 3490 AR(L,NDP1)=SYM(L,2)
      AR(L,NDPN)=SYM(L,4)
 3600 CONTINUE
 4000 NTEST=3
C
C     WRITE BLOCK OF AR, B ARRAY VALUES ONTO TAPE
C
      WRITE(34)(B(N),(AR(N,M),M=1,NDPN),N=NNP1,NN2)
      NTEST=4
      NUMBLK=NUMBLK+1
      NCNT=NUMBLK*NN
      IF(NUMBLK.EQ.1)GOTO 4300
C
C     CHECK MATRIX SYMMETRY
C
 4100 DO 4200 L=NCK1,NCK4
      NCK5=NCK5+1
      NCK2=L+1
      NCK3=L+NUMX
      IF (SYM(L,2).NE.SYM(NCK2,1)) GOTO 4150
      IF(NCK5.GT.NCK7)GOTO 4200
      IF (SYM(L,4).EQ.SYM(NCK3,3)) GOTO 4200
 4150 PRINT  17
      PRINT  23,NUMBLK,NCK5,SYM(NCK2,1),SYM(L,2),SYM(NCK3,3),SYM(L,4)
 4200 CONTINUE
      IF(NCK1.EQ.NNP1)GOTO 4500
 4300 IF(NCNT.LT.NPTS)GOTO 900
      NCK1=NNP1
      NCK4=NNP1+NPTS-NCK5-1
      GOTO 4100
 4500 CONTINUE
      PRINT  15,NCK5
      NCK8=NPTS-(NUMBLK-1)*NN
      PRINT  2 ,NCK8
C
C     ZERO AR AND B ARRAYS
C
      DO 4800 J=1,NN2
      B(J)=0.0
      DO 4800 L=1,NN
 4800 AR(J,L)=0.0
C
```

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```
C     CALL BANDED MATRIX SOLVER
C
      M=1
      IF (HX(1).NE.0.0) M=2
      CALL BANSOL
      NTEST=5
      PRINT  16,NTEST
      PRINT  2 ,LX,LY
C
C     WRITE SOLUTION ONTO TAPE
C
      WRITE(10,2)   NX,NY
      WRITE(10,4)   (HX(J),J=1,NX),(HY(K),K=1,NY)
      WRITE(10,4)   ((V(I,J),I=1,NX),J=1,NY)
      END FILE10
C
C     PRINT SOLUTION
C
 5000 M2=0
      M1=50
 5100 M2=M2+50
      IF (M2.LE.NY) GOTO 5200
      M1=NY+50-M2
      M2=NY
 5200 N2=0
 5300 N1=N2+1
      N2=N2+14
      IF (N2.GT.NX) N2=NX
      PRINT  8 ,(HX(K),K=N1,N2)
      PRINT  10
      DO 5400 J=1,M1
      I=M2+1-J
 5400 PRINT  11,HY(I),(V(K,I),K=N1,N2)
      PRINT  10
      PRINT  9 ,(HX(K),K=N1,N2)
      IF (N2.LT.NX) GOTO 5300
      IF (M2.LT.NY) GOTO 5100
      STOP
      END
      SUBROUTINE BANSOL
C
C     SOLVES BANDED, SYMMETRIC MATRIX
C
      COMMON /BANARG/ MM,NUMBLK,B,A,NN,NN2,V,NX,NY,LX,LY,LHB,NCKB
      DIMENSION B(200),A(200,100),V(100,200)
      NL=NN+1
      NH=NN+NN
      LX=NX
      LY=NY-1
      REWIND 34
      NB=0
      GO TO 150
C*********************************************************************
C     REDUCE EQUATIONS BY BLOCKS
C*********************************************************************
C
C     1. SHIFT BLOCK OF EQUATIONS
```

```
C
  100 NB=NB+1
      DO 125 N=1,NN
      NM=NN+N
      B(N)=B(NM)
      B(NM)=0.0
      DO 125 M=1,MM
      A(N,M)=A(NM,M)
  125 A(NM,M)=0.0
C
C     2. READ NEXT BLOCK OF EQUATIONS INTO CORE
C
      IF (NUMBLK-NB) 150,200,150
  150 READ (34) (B(N),(A(N,M),M=1,MM),N=NL,NH)
      IF (NB) 200,100,200
C
C     3. REDUCE BLOCK OF EQUATIONS
C
  200 DO 300 N=1,NN
      IF (A(N,1)) 225,300,225
  225 B(N)=B(N)/A(N,1)
      DO 275 L=2,MM
      IF (A(N,L)) 230,275,230
  230 C=A(N,L)/A(N,1)
      I=N+L-1
      J=0
      DO 250 K=L,MM
      J=J+1
  250 A(I,J)=A(I,J)-C*A(N,K)
      B(I)=B(I)-A(N,L)*B(N)
      A(N,L)=C
  275 CONTINUE
  300 CONTINUE
C
C     4. WRITE BLOCK OF REDUCED EQUATIONS ON TAPE 2
C
      IF (NUMBLK-NB) 375,400,375
  375 WRITE (33)(B(N),(A(N,M),M=2,MM),N=1,NN)
      GO TO 100
C***********************************************************************
C     BACK-SUBSTITUTION
C***********************************************************************
  400 DO 450 M=1,NN
      N=NN+1-M
      DO 425 K=2,MM
      L=N+K-1
  425 B(N)=B(N)-A(N,K)*B(L)
      NM=N+NN
      B(NM)=B(N)
      IF (NB.EQ.NUMBLK.AND.N.GT.NCK8) GOTO 450
      LX=LX-1
      IF (LX.GE.LHB) GOTO 445
      LY=LY-1
      LX=NX-1
  445 V(LX,LY)=B(N)
  450 CONTINUE
      NB=NB-1
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```
      IF (NB) 475,500,475
  475 BACKSPACE  33
      READ (33)  (B(N),(A(N,M),M=2,MM),N=1,NN)
      BACKSPACE  33
      GO TO 400
  500 CONTINUE
C
      RETURN
C
      END
      SUBROUTINE HORIZ(LY,J,EPS1,RA,RB,ANEPS,AREA,LX,R)
C
C     CALCULATES DIELECTRIC-AREA PRODUCT
C     OF HORIZONTAL FACE OF VOLUME ELEMENT
C
      DIMENSION EPS(199,10)
      INTEGER BDR(199,10)
      COMMON/HORARG/BDR,EPS
      IF (BDR(LY,J)-2*LX) 1,2,3
    1 J=J+1
      EPS1=EPS(LY,J)
      IF (BDR(LY,J)-2*LX) 1,2,3
    2 J=J+1
      EPS2=EPS(LY,J)
      ANEPS=0.5*((EPS1-EPS2)*R**2+EPS2*RB**2-EPS1*RA**2)
      EPS1=EPS2
      RETURN
    3 ANEPS=EPS1*AREA
      RETURN
      END
      SUBROUTINE VERT(LX,EPS1,Z,ZPH,R,AZEPS,LY,Y)
C
C     CALCULATES DIELECTRIC-AREA PRODUCT
C     OF VERTICAL FACE OF VOLUME ELEMENT
C
      DIMENSION EPS(99,10)
      INTEGER BDZ(99,10)
      COMMON/VERARG/BDZ,EPS
      K=1
      IF (BDZ(LX,K)-2*LY) 1,2,3
    1 K=K+1
      EPS1=EPS(LX,K)
      IF (BDZ(LX,K)-2*LY) 1,2,3
    2 K=K+1
      EPS2=EPS(LX,K)
      AZEPS=R*(EPS1*(Y-Z)+EPS2*(ZPH-Y))
      EPS1=EPS2
      RETURN
    3 AZEPS=EPS1*R*(ZPH-Z)
      RETURN
      END
      SUBROUTINE SPCOEF (N)
C
C     CALCULATES COEFICIENTS OF CUBIC SPLINE USED IN INTERPOLATION
C
      DIMENSION XN(200),FN(200),S(200),RHO(200),TAU(200)
      COMMON/SPARG/XN,FN,S
```

```
      NM1=N-1
      NM2=N-2
      RHO(2)=0.0
      TAU(2)=0.0
      DO 1 I=2,NM1
      IM1=I-1
      IP1=I+1
      HIM1=XN(I)-XN(IM1)
      HI=XN(IP1)-XN(I)
      TEMP=(HIM1/HI)*(RHO(I)+2.)+2.
      RHO(I+1)=-1./TEMP
      D=6.*((FN(IP1)-FN(I))/HI-(FN(I)-FN(IM1))/HIM1)/HI
    1 TAU(I+1)=(D-HIM1*TAU(I)/HI)/TEMP
      S(1)=0.
      S(N)=0.
      DO 2 I=1,NM2
      IB=N-I
    2 S(IB)=RHO(IB+1)*S(IB+1)+TAU(IB+1)
      RETURN
      END
      FUNCTION SPLINE (N,X)
C
C     EVALUATES SPLINE FUNCTION
C
      DIMENSION XN(200),FN(200),S(200)
      COMMON/SPARG/XN,FN,S
      IF (X.GE.XN(1)) GOTO 1
      H1=XN(2)-XN(1)
      SPLINE=FN(1)+(X-XN(1))*((FN(2)-FN(1))/H1-H1*S(2)/6.)
      RETURN
    1 IF (X.LE.XN(N)) GOTO 2
      NM1=N-1
      HNM1=XN(N)-XN(NM1)
      SPLINE=FN(N)+(X-XN(N))*((FN(N)-FN(NM1))/HNM1+HNM1*S(NM1)/6.)
      RETURN
    2 DO 3 I=2,N
      IF (X.LE.XN(I)) GOTO 4
    3 CONTINUE
    4 L=I-1
      LP1=L+1
      A=XN(LP1)-X
      B=X-XN(L)
      HL=XN(LP1)-XN(L)
      SPLINE=A*S(L)*(A**2/HL-HL)/6.+B*S(L+1)*(B**2/HL-HL)/6.
     /  +(A*FN(L)+B*FN(LP1))/HL
      RETURN
      END
```

FVPLOT

```
C
C     CONTOUR LINE PLOTTER
C
      DIMENSION V(100,200),HX(100),HY(200),IBUF(60),GX(100),GY(200),
     /U(100,200),LX(150),LY(150)
      COMMON/DRAWARG/X(600),Y(600),XMAX,YMAX,NQUAD,FIRSTX,DELTAX,FIRSTY,
     /DELTAY
    1 FORMAT(8F10.3)
    2 FORMAT (20I4)
    3 FORMAT(6I4,F5.0)
    4 FORMAT(I4,F10.0)
    5 FORMAT (5H1NX =,I4,6X,4HNY =,I4)
    6 FORMAT (1H0,I13,I14,I24,I14)
    7 FORMAT (7H0X FROM,F10.3,4H  TO,F10.3,8X,6HY FROM,F10.3,4H  TO,
     1F10.3)
C
C     READ POTENTIAL GRID DATA
C
      READ  4 ,IVU,UPOT
      READ  3 ,IO,IN,JO,JN,NQUAD,KK,SZ
      READ      3,   NX,NY
      READ      1,   (HX(I),  I=1,NX), (HY(J), J=1,NY)
      READ      1,   ((V(I,J), I=1,NX), J=1,NY)
      ISCAN=1
      IF (IVU.EQ.1) GOTO 150
C
C     READ FLUX GRID DATA
C
      READ  3 ,MH,NH,KRA
      READ      3,   MX,MY
      READ      1,   (GX(I),  I=1,MX), (GY(J), J=1,MY)
      READ      1,   ((U(I,J), I=1,MX), J=1,MY)
C
C     SHIFT DATA FOR REGION TO BE PLOTTED
C
      IF (IO.EQ.1) GOTO 120
      DO 100 I=IO,IN
      GX(I+1-IO)=GX(I)
      DO 100 J=1,MY
  100 U(I+1-IO,J)=U(I,J)
  120 MX=IN+1-IO
      IF (JO.EQ.1) GOTO 150
      DO 140 J=JO,JN
      GY(J+1-JO)=GY(J)
      DO 140 I=1,MX
  140 U(I,J+1-JO)=U(I,J)
  150 IF (IO.EQ.1) GOTO 170
      DO 160 I=IO,IN
      HX(I+1-IO)=HX(I)
      DO 160 J=1,NY
  160 V(I+1-IO,J)=V(I,J)
  170 NX=IN+1-IO
      IF (JO.EQ.1) GOTO 200
      DO 180 J=JO,JN
      HY(J+1-JO)=HY(J)
      DO 180 I=1,NX
  180 V(I,J+1-JO)=V(I,J)
```

```
  200 NY=JN+1-JO
      PRINT  5 ,NX,NY
      PRINT  6 ,IO,IN,JO,JN
      PRINT  7 ,HX(1),HX(NX),HY(1),HY(NY)
      XMAX=HX(NX)-HX(1)
      YMAX=HY(NY)-HY(1)
      KS=0
      NXM1=NX-1
      NYM1=NY-1
C
C     INITIALIZE PLOT AND DRAW AXES
C
      CALL PLOTS (11,0,99)
      CALL FACTOR(0.25)
      CALL PLOT (0.,-14.,-3)
      CALL PLOT(0.,0.5,-3)
      FIRSTX=HX(1)
      FIRSTY=HY(1)
      IF (XMAX.GT.YMAX) GOTO 220
      IF (NQUAD.EQ.2.AND.(2.*XMAX).GT.YMAX) GOTO 220
      DELTAX=XMAX/10.
      DELTAY=DELTAX
      GOTO 240
  220 DELTAY=YMAX/10.
      DELTAX=DELTAY
      IF (NQUAD.EQ.2) GOTO 290
  240 IF (NQUAD.GT.1) GOTO 280
      IF (XMAX.GT.YMAX) GOTO 260
      YAX=(YMAX/XMAX)*10.
      CALL AXIS(0.,10.,27HZ-AXIS      AXIS OF SYMMETRY,+27,YAX,0.,FIRSTY,
     1DELTAY,0)
      CALL AXIS(0.,10.,6HR-AXIS,-6,10.,270.,FIRSTX,DELTAX,0)
      GOTO 290
  260 XAX=(XMAX/YMAX)*10.
      CALL AXIS(0.,0.,27HZ-AXIS      AXIS OF SYMMETRY,+27,10.,90.,FIRSTY,
     1DELTAY,0)
      CALL AXIS(0.,0.,6HR-AXIS,-6,XAX,0.,FIRSTX,DELTAX,0)
      GOTO 290
  280 DELTAX=2.*DELTAX
      DELTAY=2.*DELTAY
  290 IF (IVU.EQ.2) GOTO 305
C
C     DRAW EQUIPOTENTIAL LINES
C
      DO 300 K=1,KK
      CZ=K-KS
      Z=CZ*SZ
  300 CALL DRAW (Z,NX,NY,NXM1,NYM1,HX,HY,V,ISCAN)
      IF (IVU.EQ.3) 305,500
C
C     CALCULATE AND PLOT FLUX LINE DISTRIBUTION
C
  305 MXM1=MX-1
      MYM1=MY-1
      I=1
      PA=V(1,1)
  310 I=I+1
```

```
      PB=V(1,I)
      IF (UPOT.GE.PA.AND.UPOT.LE.PB) GOTO 320
      PA=PB
      GOTO 310
  320 IM=I-1
      R=HY(IM)+(UPOT-PA)*(HY(I)-HY(IM))/(PB-PA)
      H=HY(NH)-HY(MH)
      ISCAN=2
      C=0.
      RA=HX(KRA)
  330 KRB=KRA+1
      IF (KRB.GT.NX)  GOTO 500
      RB=HX(KRB)
      EA=(V(KRA,NH)-V(KRA,MH))/H
      EB=(V(KRB,NH)-V(KRB,MH))/H
      A2=((EB-EA)/(RB-RA))/2.
      A1=EA-RA*2.*A2
      CINC=A2*(RB**2-RA**2)+A1*(RB-RA)
      IF (R.LT.RB) CINC=A2*(R**2-RA**2)+A1*(R-RA)
      C=C+CINC
      IF (R.LE.RB) GOTO 340
      KRA=KRB
      RA=RB
      GOTO 330
  340 Z=U(KRA,MH)+(R-RA)*(U(KRB,MH)-U(KRA,MH))/(RB-RA)
      CALL DRAW (Z,NX,NY,NXM1,NYM1,GX,GY,U,ISCAN)
  350 RX=R
      CSUM=0.
  360 A2=((EB-EA)/(RB-RA))/2.
      A1=EA-RA*2.*A2
      CINC=A2*(RB**2-RX**2)+A1*(RB-RX)
      IF (C-CSUM-CINC) 380,390,370
  370 CSUM=CSUM+CINC
      KRA=KRB
      KRB=KRA+1
      IF (KRB.GT.NX)  GOTO 500
      RA=RB
      RX=RA
      RB=HX(KRB)
      EA=(V(KRA,NH)-V(KRA,MH))/H
      EB=(V(KRB,NH)-V(KRB,MH))/H
      GOTO 360
  380 A0=CSUM-(C+A2*RX**2+A1*RX)
      CALL RSOLV(R,RX,RB,A2,A1,A0)
      GOTO 400
  390 R=RB
  400 Z=U(KRA,MH)+(R-RA)*(U(KRB,MH)-U(KRA,MH))/(RB-RA)
      CALL DRAW (Z,NX,NY,NXM1,NYM1,GX,GY,U,ISCAN)
      IF (ISCAN.EQ.0) GOTO 500
      IF (R.NE.RB) GOTO 350
      KRA=KRB
      KRB=KRB+1
      IF (KRB.GT.NX)  GOTO 500
      RB=HX(KRB)
      RA=R
      GOTO 350
C
```

```
C     DRAW ELECTRODE AND DIELECTRIC CONFIGURATION
C
  500 READ  3 ,NLINES
      DO 700 L=1,NLINES
      READ  3 ,NPTS
      READ  2 ,(LX(N),LY(N),N=1,NPTS)
      IF (XMAX.GT.YMAX) GOTO 600
      IF (NQUAD.EQ.2.AND.(2.*XMAX).GT.YMAX) GOTO 600
      DO 540 N=1,NPTS
      LXNNN=LX(N)+1-IO
      LYNNN=LY(N)+1-JO
      Y(N)=HY(LYNNN)
      GOTO (510,530,520,520),NQUAD
  510 X(N)=XMAX-HX(LXNNN)
      GOTO 540
  520 Y(N)=YMAX-HY(LYNNN)
  530 X(N)=XMAX+HX(LXNNN)
  540 CONTINUE
      X(NPTS+1)=FIRSTX
      X(NPTS+2)=DELTAX
      Y(NPTS+1)=FIRSTY
      Y(NPTS+2)=DELTAY
      CALL LINE (Y,X,NPTS,1,0,0)
      GOTO (700,580,550,550),NQUAD
  550 DO 560 N=1,NPTS
  560 X(N)=2.*XMAX-X(N)
      CALL LINE (Y,X,NPTS,1,0,0)
      DO 570 N=1,NPTS
  570 Y(N)=2.*YMAX-Y(N)
      CALL LINE (Y,X,NPTS,1,0,0)
  580 DO 590 N=1,NPTS
  590 X(N)=2.*XMAX-X(N)
      CALL LINE (Y,X,NPTS,1,0,0)
      GOTO 700
  600 DO 640 N=1,NPTS
      LXNNN=LX(N)+1-IO
      LYNNN=LY(N)+1-JO
      Y(N)=HY(LYNNN)
      GOTO (610,630,620,620),NQUAD
  610 X(N)=HX(LXNNN)
      GOTO 640
  620 Y(N)=YMAX-HY(LYNNN)
  630 X(N)=XMAX+HX(LXNNN)
  640 CONTINUE
      X(NPTS+1)=FIRSTX
      X(NPTS+2)=DELTAX
      Y(NPTS+1)=FIRSTY
      Y(NPTS+2)=DELTAY
      CALL LINE (X,Y,NPTS,1,0,0)
      GOTO (700,680,650,650),NQUAD
  650 DO 660 N=1,NPTS
  660 X(N)=2.*XMAX-X(N)
      CALL LINE (X,Y,NPTS,1,0,0)
      DO 670 N=1,NPTS
  670 Y(N)=2.*YMAX-Y(N)
      CALL LINE (X,Y,NPTS,1,0,0)
  680 DO 690 N=1,NPTS
```

```
690 X(N)=2.*XMAX-X(N)
    CALL LINE (X,Y,NPIS,1,0,0)
700 CONTINUE
    CALL PLOT(12.0,0.0,40)
    STOP
    END
    SUBROUTINE DRAW (Z,IL,JL,II,JJ,XV,YV,V,ISCAN)
C
C     SCANS GRID AND PLOTS LINES OF EQUAL VALUE
C
    DIMENSION XV(100),YV(200),V(100,200)
    COMMON/DRAWARG/X(600),Y(600),XMAX,YMAX,NQUAD,FIRSTX,DELTAX,FIRSTY,
   /DELTAY
    NT=0
    IX=0
    T=Z
    IA=IL
    DO 3 J=1,JJ
    JA=J
    A=V(IA,JA)
    D=V(IA,JA+1)
    IF (A.EQ.D) GO TO 3
    IF (A.LE.0..OR.D.LE.0.) GOTO 3
    IF (A.EQ.T) 1,2
  1 T=A+(D-A)*1.E-6
  2 CALL DA(D,A,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
    IF (NT.NE.0) GO TO 30
  3 CONTINUE
    JA=JL
    DO 6 I=1,II
    IA=IL-I
    B=V(IA+1,JA)
    A=V(IA,JA)
    IF (B.EQ.A) GO TO 6
    IF (A.LE.0..OR.B.LE.0.) GOTO 6
    IF (B.EQ.T) 4,5
  4 T=B+(A-B)*1.E-6
  5 CALL AB(A,B,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
    IF (NT.NE.0) GO TO 30
  6 CONTINUE
    IA=0
    DO 10 J=1,JJ
    JA=JL-J
    C=V(1,JA+1)
    B=V(1,JA)
    IF (C.EQ.B) GO TO 10
    IF (JA.EQ.1) GOTO 7
    IF (C.LE.0..OR.B.LE.0.) GOTO 10
  7 IF (C.LE.0..OR.B.LT.0.) GOTO 10
    IF (C.EQ.T) 8,9
  8 T=C+(B-C)*1.E-6
  9 CALL BC(B,C,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
    IF (NT.NE.0) GO TO 30
 10 CONTINUE
    JA=0
    DO 14 I=1,II
    IA=I
```

```
      D=V(IA,1)
      C=V(IA+1,1)
      IF (D.EQ.C) GO TO 14
      IF (IA.EQ.1.AND.ISCAN.EQ.2) GOTO 11
      IF (C.LE.0..OR.D.LE.0.) GOTO 14
   11 IF (C.LE.0..OR.D.LT.0.) GOTO 14
      IF (D.EQ.T) 12,13
   12 T=D+(C-D)*1.E-6
   13 CALL CD(C,D,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
      IF (NT.NE.0) GO TO 30
   14 CONTINUE
      IF (ISCAN.NE.1) GOTO 22
      DO 21 I=1,II
      DO 21 J=1,JJ
      IA=I
      JA=J
      C=V(I+1,J+1)
      B=V(I+1,J)
      IF (C.EQ.B) GO TO 18
      IF (J.EQ.1) GOTO 15
      IF (C.LE.0..OR.B.LE.0.) GOTO 18
   15 IF (C.LE.0..OR.B.LT.0.) GOTO 18
      IF (C.EQ.T) 16,17
   16 T=C+(B-C)*1.E-6
   17 CALL BC(B,C,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
      IF (NT.NE.0) GO TO 30
   18 D=V(I,J+1)
      IF (C.EQ.D) GO TO 21
      IF (C.LE.0..OR.D.LE.0.) GOTO 21
      IF (C.EQ.T) 19,20
   19 T=C+(D-C)*1.E-6
   20 CALL CD(C,D,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
      IF (NT.NE.0) GO TO 30
   21 CONTINUE
      ISCAN=0
      GO TO 69
   22 DO 29 J=1,JJ
      DO 29 I=1,II
      IA=I
      JA=J
      C=V(I+1,J+1)
      B=V(I+1,J)
      IF (C.EQ.B) GO TO 25
      IF (C.LE.0..OR.B.LE.0.) GOTO 25
      IF (C.EQ.T) 23,24
   23 T=C+(B-C)*1.E-6
   24 CALL BC(B,C,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
      IF (NT.NE.0) GO TO 30
   25 D=V(I,J+1)
      IF (C.EQ.D) GO TO 29
      IF (I.EQ.1) GOTO 26
      IF (C.LE.0..OR.D.LE.0.) GOTO 29
   26 IF (C.LE.0..OR.D.LT.0.) GOTO 29
      IF (C.EQ.T) 27,28
   27 T=C+(D-C)*1.E-6
   28 CALL CD(C,D,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
      IF (NT.NE.0) GO TO 30
```

```
29 CONTINUE
   ISCAN=0
   GO TO 69
30 IST=IA
   JST=JA
31 IF (IA.EQ.0.OR.IA.EQ.IL.OR.JA.EQ.0.OR.JA.EQ.JL) GO TO 50
   NIX=IX
   IF (T.EQ.V(IA,JA))  V(IA,JA)=V(IA,JA)*1.0001
   IF (T.EQ.V(IA+1,JA)) V(IA+1,JA)=V(IA+1,JA)*1.0001
   IF (T.EQ.V(IA+1,JA+1)) V(IA+1,JA+1)=V(IA+1,JA+1)*1.0001
   IF (T.EQ.V(IA,JA+1)) V(IA,JA+1)=V(IA,JA+1)*1.0001
   A=V(IA,JA)
   B=V(IA+1,JA)
   C=V(IA+1,JA+1)
   D=V(IA,JA+1)
   IF (IA.EQ.1.AND.ISCAN.EQ.2) GOTO 32
   IF (JA.EQ.1) GOTO 33
   IF (A.LE.0..OR.B.LE.0..OR.C.LE.0..OR.D.LE.0.) GOTO 40
32 IF (A.LT.0..OR.B.LE.0..OR.C.LE.0..OR.D.LT.0.) GOTO 50
   GOTO 34
33 IF (A.LT.0..OR.B.LT.0..OR.C.LE.0..OR.D.LE.0.) GOTO 50
34 IF (NT.EQ.1) GO TO 35
   IF (NT.EQ.2) GO TO 36
   IF (NT.EQ.3) GO TO 37
   IF (NT.EQ.4) GO TO 38
   GO TO 69
35 CALL DA(D,A,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
   IF (IX.GT.NIX) GOTO 39
   CALL BC(B,C,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
   IF (IX.GT.NIX) GOTO 39
   CALL AB(A,B,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
   GO TO 39
36 CALL AB(A,B,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
   IF (IX.GT.NIX) GOTO 39
   CALL CD(C,D,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
   IF (IX.GT.NIX) GOTO 39
   CALL BC(B,C,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
   GO TO 39
37 CALL BC(B,C,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
   IF (IX.GT.NIX) GOTO 39
   CALL DA(D,A,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
   IF (IX.GT.NIX) GOTO 39
   CALL CD(C,D,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
   GO TO 39
38 CALL CD(C,D,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
   IF (IX.GT.NIX) GOTO 39
   CALL AB(A,B,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
   IF (IX.GT.NIX) GOTO 39
   CALL DA(D,A,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
39 IF (IA.EQ.IST.AND.JA.EQ.JST) 50,31
40 IF (X(IX).EQ.X(IX-1)) GOTO 50
   S=(Y(IX)-Y(IX-1))/(X(IX)-X(IX-1))
   SB=Y(IX)-S*X(IX)
41 SL=(YV(JA+1)-YV(JA))/(XV(IA)-XV(IA+1))
   SR=(YV(JA+1)-YV(JA))/(XV(IA+1)-XV(IA))
   ORTHL=ABS(SL+1./S)
   ORTHR=ABS(SR+1./S)
```

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      IF (ORTHR.LT.ORTHL) GOTO 42
      SBD=YV(JA)-SL*XV(IA+1)
      XR=(SBD-SB)/(S-SL)
      GOTO 43
   42 SBD=YV(JA)-SR*XV(IA)
      XR=(SBD-SB)/(S-SR)
   43 IX=IX+1
      X(IX)=XR
      Y(IX)=S*X(IX)+SB
   50 CALL XPLOTY(X,Y,Z,IX)
      IF (XMAX.GT.YMAX) GOTO 60
      IF (NQUAD.EQ.2.AND.(2.*XMAX).GT.YMAX) GOTO 60
      DO 54 I=1,IX
      GOTO (51,53,52,52),NQUAD
   51 X(I)=XMAX-X(I)
      GOTO 54
   52 Y(I)=YMAX-Y(I)
   53 X(I)=XMAX+X(I)
   54 CONTINUE
      X(IX+1)=FIRSTX
      X(IX+2)=DELTAX
      Y(IX+1)=FIRSTY
      Y(IX+2)=DELTAY
      CALL LINE(Y,X,IX,1,0,0)
      GOTO (69,58,55,55),NQUAD
   55 DO 56 I=1,IX
   56 X(I)=2.*XMAX-X(I)
      CALL LINE(Y,X,IX,1,0,0)
      DO 57 I=1,IX
   57 Y(I)=2.*YMAX-Y(I)
      CALL LINE(Y,X,IX,1,0,0)
   58 DO 59 I=1,IX
   59 X(I)=2.*XMAX-X(I)
      CALL LINE(Y,X,IX,1,0,0)
      GOTO 69
   60 GOTO (63,61,61,61),NQUAD
   61 DO 62 I=1,IX
      X(I)=XMAX+X(I)
   62 IF (NQUAD.EQ.4) Y(I)=2.*YMAX-Y(I)
   63 X(IX+1)=FIRSTX
      X(IX+2)=DELTAX
      Y(IX+1)=FIRSTY
      Y(IX+2)=DELTAY
      CALL LINE(X,Y,IX,1,0,0)
      GOTO (69,67,64,64),NQUAD
   64 DO 65 I=1,IX
   65 X(I)=2.*XMAX-X(I)
      CALL LINE(X,Y,IX,1,0,0)
      DO 66 I=1,IX
   66 Y(I)=2.*YMAX-Y(I)
      CALL LINE(X,Y,IX,1,0,0)
   67 DO 68 I=1,IX
   68 X(I)=2.*XMAX-X(I)
      CALL LINE(X,Y,IX,1,0,0)
   69 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RSOLV (R,RA,RB,A2,A1,A0)
C
C     CALCULATES ROOTS OF QUADRATIC EQUATION
C
      R=(-A1 - SQRT(A1**2 - 4.*A2*A0)) /(2.*A2)
      PRINT 1 ,R,RA,RB,A2,A1,A0
    1 FORMAT (4H R =,6E16.8)
      IF (R.GE.RA.AND.R.LE.RB) GOTO 2
      R=(-A1 + SQRT(A1**2 - 4.*A2*A0)) /(2.*A2)
      PRINT 1 ,R,RA,RB,A2,A1,A0
    2 CONTINUE
      RETURN
      END
      SUBROUTINE XPLOTY(X,Y,Z,IX)
C
C     PRINTS COORDINATES OF EQUAL VALUE LINES
C
      DIMENSION X(1),Y(1)
      PRINT 3
      PRINT 2, IX
      PRINT 4, Z
      PRINT 1,  (X(K), K=1,IX)
      PRINT 3
      PRINT 1,  (Y(K), K=1,IX)
      PRINT 3
 1    FORMAT(1H ,8E16.8)
 2    FORMAT(1H ,4I6,7H POINTS)
 3    FORMAT(1H )
 4    FORMAT(9H VALUE = ,F8.3)
      RETURN
      END
      SUBROUTINE AB(A,B,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
C
C     SEARCHES BOTTOM SIDE OF RECTANGLE FOR GIVEN VALUE
C
      DIMENSION XV(1),YV(1),X(1),Y(1),V(100,200)
      IF (A.GT.T.AND.B.LT.T.OR.A.LT.T.AND.B.GT.T) 1,7
 1    P1=XV(IA)
      Q1=XV(IA+1)
      XX=(P1+Q1)/2.
      T1=A
      T2=B
      IF (IA.EQ.1) 2,3
 2    R1=XV(IA+2)
      T3=V(IA+2,JA)
      CALL SOL33(XX,T,P1,Q1,R1,T1,T2,T3)
      GO TO 6
 3    IF (IA.EQ.II) 4,5
 4    P1=XV(IA-1)
      T3=V(IA-1,JA)
      CALL SOL33(XX,T,P1,Q1,R1,T1,T2,T3)
      GO TO 6
 5    IF (V(IA-1,JA).LE.0.) GOTO 2
      IF (V(IA+2,JA).LE.0.) GOTO 4
      R1=XV(IA-1)
      S1=XV(IA+2)
      T3=V(IA-1,JA)
```

```
      T4=V(IA+2,JA)
      CALL SOL44(XX,T,P1,Q1,R1,S1,T1,T2,T3,T4)
    6 NT=1
      IX=IX+1
      X(IX)=XX
      Y(IX)=YV(JA)
      JA=JA-1
    7 RETURN
      END
      SUBROUTINE BC(B,C,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
C
C     SEARCHES R.H. SIDE OF RECTANGLE FOR GIVEN VALUE
C
      DIMENSION XV(1),YV(1),X(1),Y(1),V(100,200)
      IF (B.GT.T.AND.C.LT.T.OR.B.LT.T.AND.C.GT.T) 1,7
    1 P1=YV(JA)
      Q1=YV(JA+1)
      XX=(P1+Q1)/2.
      T1=B
      T2=C
      IF (JA.EQ.1) 2,3
    2 R1=YV(JA+2)
      T3=V(IA+1,JA+2)
      CALL SOL33(XX,T,P1,Q1,R1,T1,T2,T3)
      GO TO 6
    3 IF (JA.EQ.JJ) 4,5
    4 R1=YV(JA-1)
      T3=V(IA+1,JA-1)
      CALL SOL33(XX,T,P1,Q1,R1,T1,T2,T3)
      GO TO 6
    5 IF (V(IA+1,JA-1).LE.0.) GOTO 2
      IF (V(IA+1,JA+2).LE.0.) GOTO 4
      R1=YV(JA-1)
      S1=YV(JA+2)
      T3=V(IA+1,JA-1)
      T4=V(IA+1,JA+2)
      CALL SOL44(XX,T,P1,Q1,R1,S1,T1,T2,T3,T4)
    6 NT=2
      IX=IX+1
      X(IX)=XV(IA+1)
      Y(IX)=XX
      IA=IA+1
    7 RETURN
      END
      SUBROUTINE CD(C,D,T,XV,YV,X,Y,IA,JA,IX,NT,II,V)
C
C     SEARCHES TOP SIDE OF RECTANGLE FOR GIVEN VALUE
C
      DIMENSION XV(1),YV(1),X(1),Y(1),V(100,200)
      IF (C.GT.T.AND.D.LT.T.OR.C.LT.T.AND.D.GT.T) 1,7
    1 P1=XV(IA)
      Q1=XV(IA+1)
      XX=(P1+Q1)/2.
      T1=D
      T2=C
      IF (IA.EQ.1) 2,3
    2 R1=XV(IA+2)
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```
      T3=V(IA+2,JA+1)
      CALL SOL33(XX,T,P1,Q1,R1,T1,T2,T3)
      GO TO 6
    3 IF (IA.EQ.II) 4,5
    4 R1=XV(IA-1)
      T3=V(IA-1,JA+1)
      CALL SOL33(XX,T,P1,Q1,R1,T1,T2,T3)
      GO TO 6
    5 IF (V(IA-1,JA+1).LE.0.) GOTO 2
      IF (V(IA+2,JA+1).LE.0.) GOTO 4
      R1=XV(IA-1)
      S1=XV(IA+2)
      T3=V(IA-1,JA+1)
      T4=V(IA+2,JA+1)
      CALL SOL44(XX,T,P1,Q1,R1,S1,T1,T2,T3,T4)
    6 NT=3
      IX=IX+1
      X(IX)=XX
      Y(IX)=YV(JA+1)
      JA=JA+1
    7 RETURN
      END
      SUBROUTINE DA(D,A,T,XV,YV,X,Y,IA,JA,IX,NT,JJ,V)
C
C     SEARCHES L.H. SIDE OF RECTANGLE FOR GIVEN VALUE
C
      DIMENSION XV(1),YV(1),X(1),Y(1),V(100,200)
      IF (D.GT.T.AND.A.LT.T.OR.D.LT.T.AND.A.GT.T) 1,7
    1 P1=YV(JA)
      Q1=YV(JA+1)
      XX=(P1+Q1)/2.
      T1=A
      T2=D
      IF (JA.EQ.1) 2,3
    2 R1=YV(JA+2)
      T3=V(IA,JA+2)
      CALL SOL33(XX,T,P1,Q1,R1,T1,T2,T3)
      GO TO 6
    3 IF (JA.EQ.JJ) 4,5
    4 R1=YV(JA-1)
      T3=V(IA,JA-1)
      CALL SOL33(XX,T,P1,Q1,R1,T1,T2,T3)
      GO TO 6
    5 IF (V(IA,JA-1).LE.0.) GOTO 2
      IF (V(IA,JA+2).LE.0.) GOTO 4
      R1=YV(JA-1)
      S1=YV(JA+2)
      T3=V(IA,JA-1)
      T4=V(IA,JA+2)
      CALL SOL44(XX,T,P1,Q1,R1,S1,T1,T2,T3,T4)
    6 NT=4
      IX=IX+1
      X(IX)=XV(IA)
      Y(IX)=XX
      IA=IA-1
    7 RETURN
      END
```

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```
      SUBROUTINE SOL33(XX,T,P1,Q1,R1,T1,T2,T3)
C
C     POLYNOMIAL INTERPOLATION WITH 3 NODES
C
      XX=XX-P1
      X1=0.
      X2=Q1-P1
      X3=R1-P1
      PQ=X2
      XX2=X2*X2
      XX3=X3*X3
      TT=T-T1
      TT2=T2-T1
      TT3=T3-T1
      DT=XX2*X3-XX3*X2
      AA=(TT2*X3-TT3*X2)/DT
      BB=(XX2*TT3-XX3*TT2)/DT
      TX=-TT
      TZ=TT2-TT
    1 TY=(AA*XX+BB)*XX-TT
      TU=TX*TY
      IF (TU) 2,5,3
    2 X2=XX
      TZ=TY
      GO TO 4
    3 X1=XX
      TX=TY
    4 XU=(X1+X2)/2.
      U=ABS((XX-XU)/PQ)
      XX=XU
      IF (U.LT..005)  5,1
    5 XX=XX+P1
      RETURN
      END
      SUBROUTINE SOL44(XX,T,P1,Q1,R1,S1,T1,T2,T3,T4)
C
C     POLYNOMIAL INTERPOLATION WITH 4 NODES
C
      XX=XX-P1
      X1=0.
      X2=Q1-P1
      X3=R1-P1
      X4=S1-P1
      PQ=X2
      XX2=X2*X2
      XX3=X3*X3
      XX4=X4*X4
      XXX2=XX2*X2
      XXX3=XX3*X3
      XXX4=XX4*X4
      TT=T-T1
      TT2=T2-T1
      TT3=T3-T1
      TT4=T4-T1
      DT=DT3(XXX2,XX2,X2,XXX3,XX3,X3,XXX4,XX4,X4)
      AA=DT3(TT2,XX2,X2,TT3,XX3,X3,TT4,XX4,X4)/DT
      BB=DT3(XXX2,TT2,X2,XXX3,TT3,X3,XXX4,TT4,X4)/DT
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```
      CC=DT3(XXX2,XX2,TT2,XXX3,XX3,TT3,XXX4,XX4,TT4)/DT
      TX=-TT
      TZ=TT2-TT
    1 TY=((AA*XX+BB)*XX+CC)*XX-TT
      TU=TX*TY
      IF (TU) 2,5,3
    2 X2=XX
      TZ=TY
      GO TO 4
    3 X1=XX
      TX=TY
    4 XU=(X1+X2)/2.
      U=ABS((XX-XU)/PQ)
      XX=XU
      IF (U.LT..005)  5,1
    5 XX=XX+PI
      RETURN
      END
      FUNCTION DT3(A1,B1,C1,A2,B2,C2,A3,B3,C3)
C
C     CALCULATES DETERMINANT OF 3 BY 3 MATRIX
C
      DT3=A1*B2*C3+A2*B3*C1+A3*B1*C2-A1*B3*C2-A2*B1*C3-A3*B2*C1
      RETURN
      END
```

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